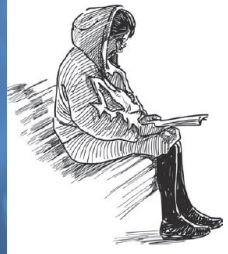
Brigham, Ehrhardt & Fox

Financial Management:
Theory and Practice
1st EMEA edition

Financial Management

Financial Management / MBAC 813

Chapter 4
Time Value of Money



Professor Ines Nel

Chapter goals

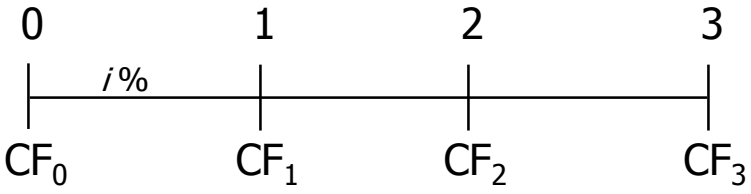
Students should be able to:

- Do time value of money calculations
  - Future value
  - Present value
  - Rate of return / Interest rate
  - Time
- for annuities, lump sums and unequal cash flows
- Make decisions base on time value of money

The role of financial markets

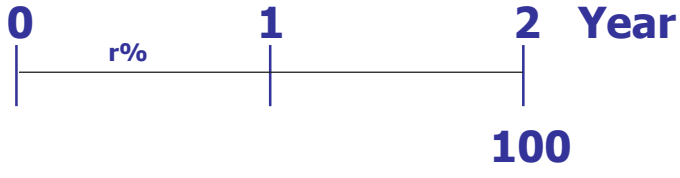
- Voluntary transfer of wealth between individuals via financial intermediates across time (FV and PV)
- Competition and financial costs
- Corporate perspective
  Required return (cost of capital, opportunity cost)
- The chance to earn a return on invested funds means a Rand today is worth more than a Rand (or dollar, euro, pound, franc, or yen) in the future

Time lines show timing of cash flows (CF).

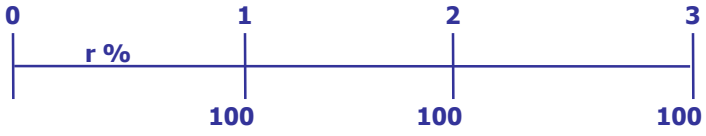


Tick marks at ends of periods, but CF_0 is today; CF_1 is the end of Period 1; CF_2 end of period 2 etc.

Time line for a R100 lump sum due at the end of Year 2.



Time line for an ordinary annuity of R100 for 3 years.



Time line for an annuity due of R100 for 3 years.

Time line for uneven CFs: -R50 at t = 0 R100, R75, and R50 at the end of Years 1 to 3.

Future Value

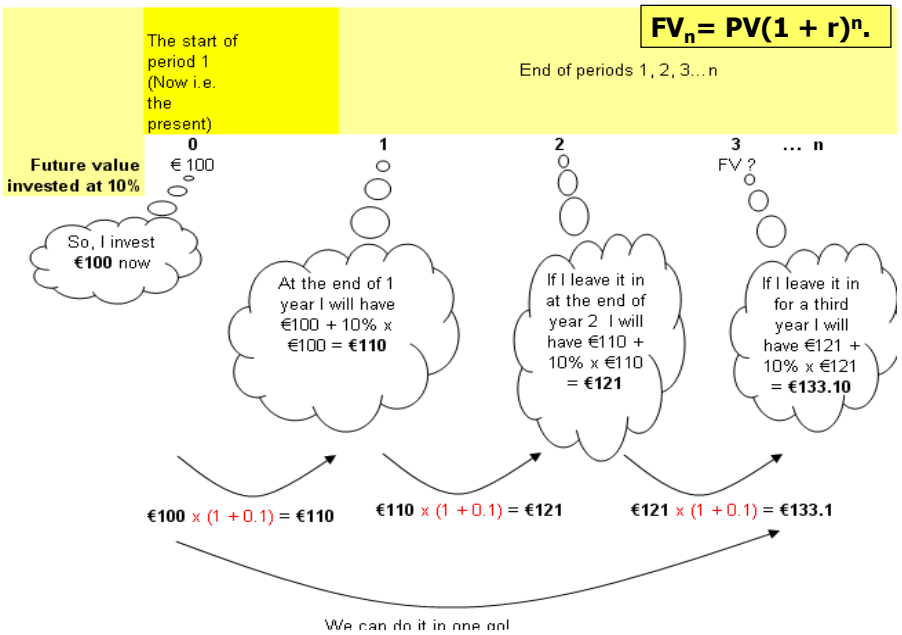
The value of a lump sum or stream of cash payments at a future point in time

$$FV_n = PV \times (1+r)^n$$

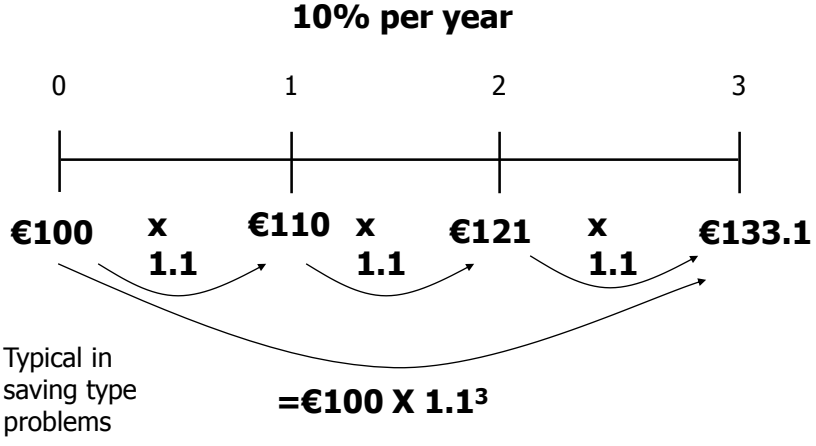
Future Value depends on:

- Interest Rate
- Number of Periods
- Compounding Interval

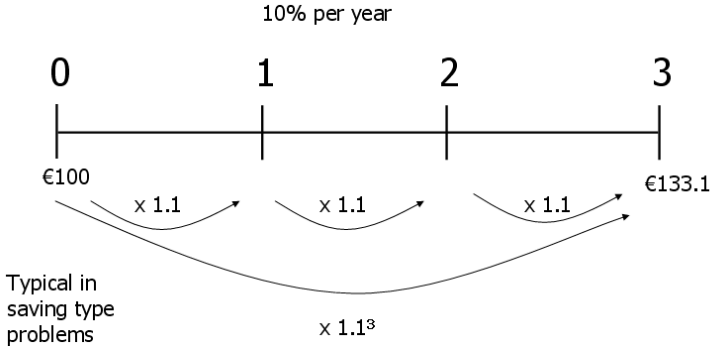
Calculating future value



Future values



Future values - the formula



$$FV_0 \text{ x } (1+i) \text{ x } (1+i) \text{ x } (1+i) = FV_3$$

$$FV_0 \text{ x } (1+i)^3 = FV_3$$

$$€100 \text{ x } 1.1^3 = €133.10$$

In general terms the future value of a single amount invested is:

$$CF_0 \text{ x } (1+i)^n = FV_n$$

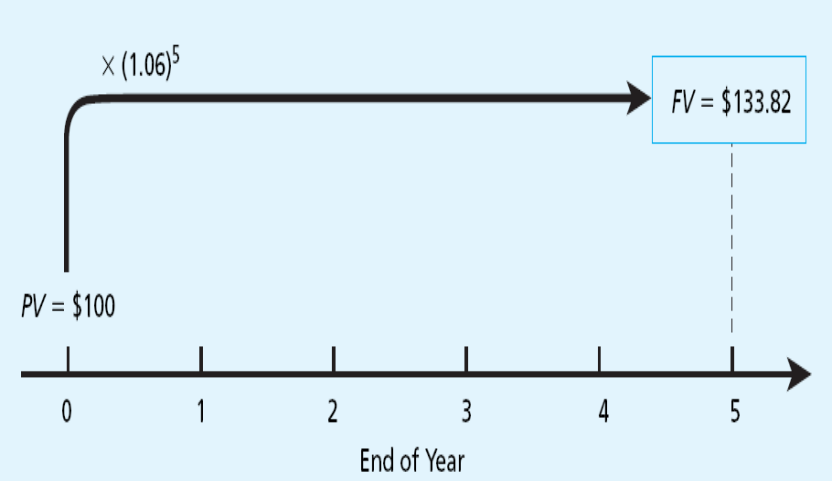
Solve $FV_n = PV(1 + i)^n$

$$PV = \frac{FV_n}{(1+i)^n} \quad \text{therefore} \quad FV_n = PV\,(1+i)^n$$

Future value of €100 invested at 10% per year = 100 x 1.10^3 = €133.1

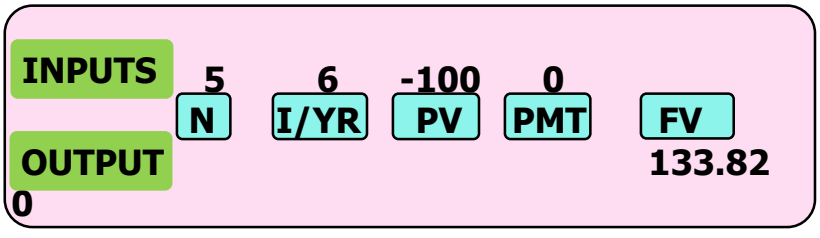
Present value of €133.1 discounted at 10% per year = 133.1 / 1.10^3 = €100

Exercise - Calculate:
Future Value of $100 (*5 Years, 6% Interest*)



Source: Megginson

Calculator solution:

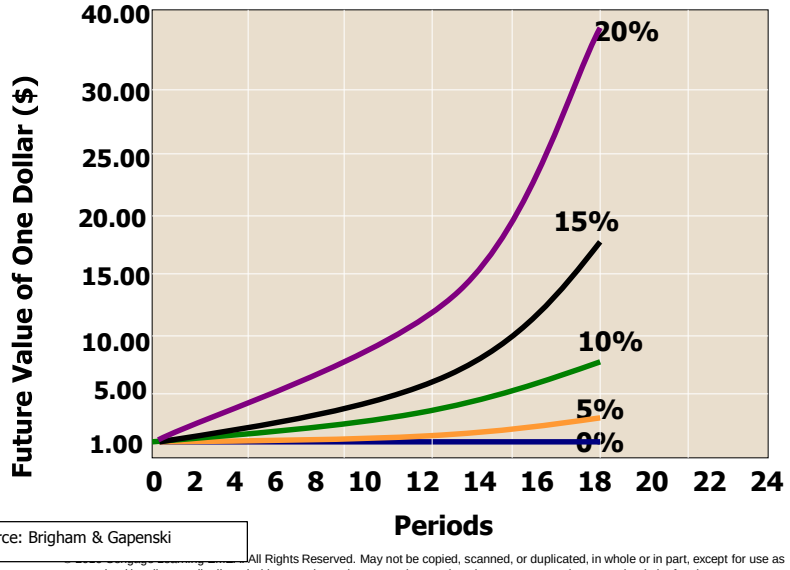


Clearing automatically sets everything to 0, but for safety enter PMT = 0.

Set: P/YR = 1, END.

Source: Brigham & Gapenski

The power of compounded interest



Source: Brigham & Gapenski

Key points:

- The higher the interest rate, the higher the future value
- The longer the period of time, the higher the future value

Compounding: Is to find the future value of present amount invested at a given rate

Present Value

Today's value of a lump sum or stream of cash payments received at a future point in time

$$FV_n = PV \times (1+r)^n$$

PV calculation is the inverse of future value calculation

$$PV = \frac{FV_n}{(1+r)^n}$$

Present Value

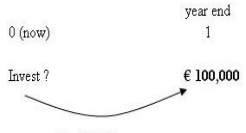
Problem : Mr A wants to raise money to fund an Arts festival in 1 year's time. How much are you willing to lend him if he offers you 20% of the net profits? Mr A estimates that he will earn €500,000

Time line :

	0 (now)	year end 1
Invest ?		0.20 x €500,000 = **€100,000**

Thoughts €100,000 is only an **expected** value. This is a very risky operation It could be a flop and earn far less, it could be a huge success and earn far more.

Calculation : I want an expected return of 25% so…



Invest? x 1.25 = €100,000

Invest? = €100,000 / 1.25

= € 80,000

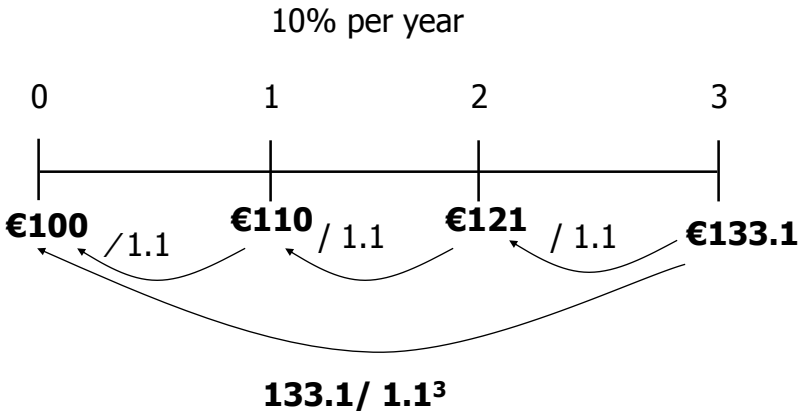
So you would be prepared to lend € 80,000 and expect a return of

$$\frac{€\,100{,}000 - €\,80{,}000}{€80000} = 0.25 \text{ or } 25\%$$

Language : "Invest?" is the **present value** of €100,000 **discounted** at 25%

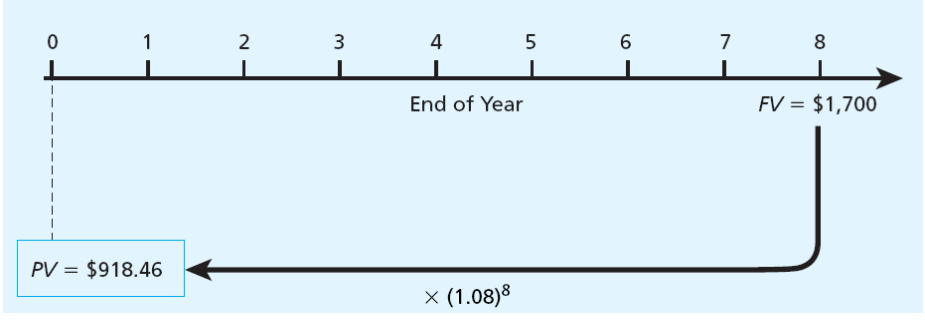
Present values

$$PV = \frac{FV_n}{(1+r)^n}$$



Typical in investment type problems

Exercise find the present value:
Future value of $1,700 (*8 Years, 8% Interest*)



Source: Megginson

Calculator Solution

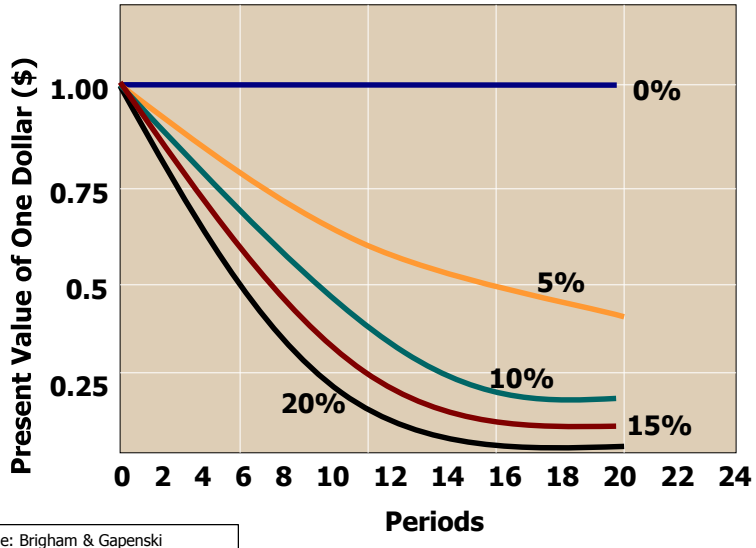
	N	I/YR	PV	PMT	FV
INPUTS	8	8		0	1700
OUTPUT			-918.46		

Clearing automatically sets everything to 0, but for safety enter PMT = 0.

Set: P/YR = 1, END.

Source: Brigham & Capenski

The power of discount rates



Source: Brigham & Gapenski

Three Ways to Find FVs

- Step-by-step approach using time line
- Solve the equation - calculator (formula approach).
- Use a spreadsheet.

Finding the Time to reach goal

$$FV = PV(1 + r)^n$$
$$R2 = R1(1 + 0.20)^n$$
$$(1.2)^n = R2/R1 = 2$$
$$nLN(1.2) = LN(2)$$
$$n = LN(2)/LN(1.2)$$
$$n = 0.693/0.182 = 3.8.$$

Finding Interest Rate that will achieve goal

$$FV = PV(1 + r)^n$$
$$R2 = R1(1 + r)^3$$
$$(2)^{(1/3)} = (1 + r)$$
$$1.2599 = (1 + r)$$
$$i = 0.2599 = 25.99\%.$$

27

FV of a mixed (uneven) cash flow stream

Future value of an *n*-year mixed stream of cash flows (*FV*) can be expressed as

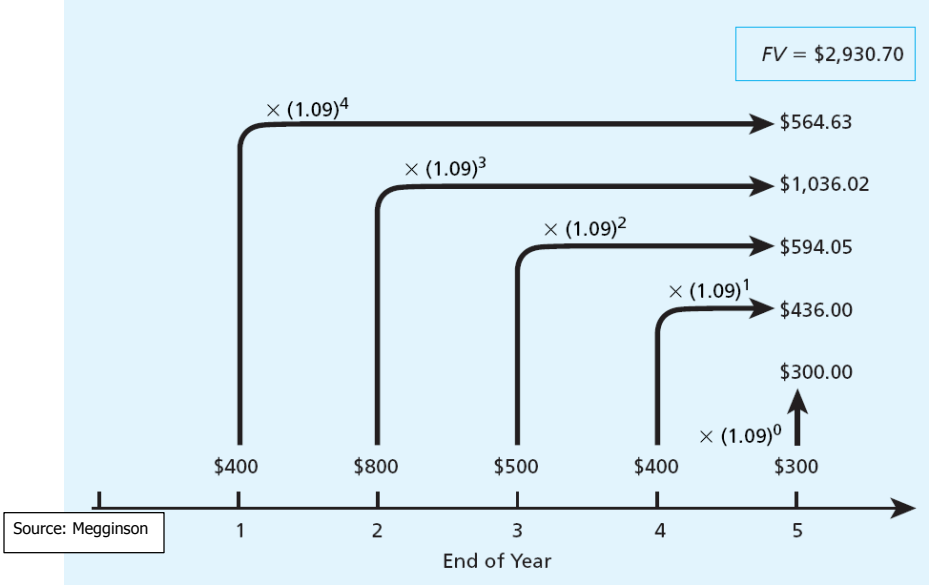
$$FV = CF_1 \times (1 + r)^{n-1} + CF_2 \times (1 + r)^{n-2} + \cdots + CF_n \times (1 + r)^{n-n}$$

or

$$FV = \sum_{t=1}^{n} CF_t \times (1 + r)^{n-t}$$

28

Exercise: Find the FV of the *mixed (uneven) cash flow stream* below invested at 9% interest



Present values of irregular cash flows

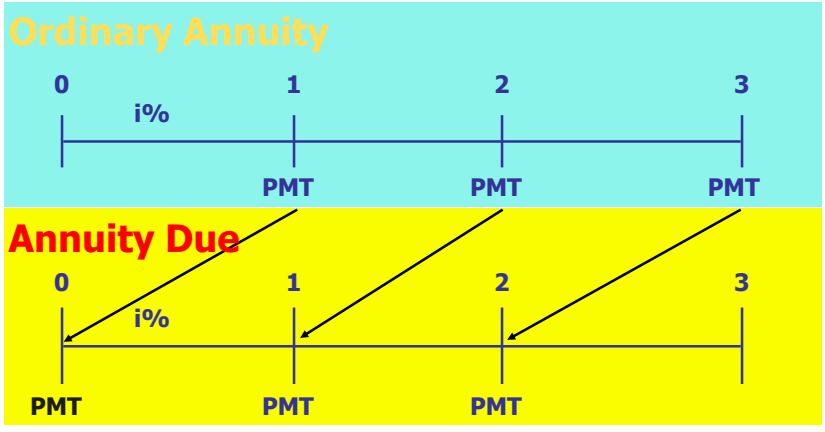
- Present value is additive you can add the present value of expected cash flows from different future time periods
- For example, a project offers *annual* cash flows of €100,150,200,250 and 250 what is the present value given an interest rate of 25%?

$$463 = \frac{100}{(1.25)^1} + \frac{150}{(1.25)^2} + \frac{200}{(1.25)^3} + \frac{250}{(1.25)^4} + \frac{250}{(1.25)^5}$$

- An investment of €463 yields a return of 25%

What is an annuity

Stream of equal cash flows at regular intervals



Annuities

Common in finance regular payments and repayments e.g.€100 per month for 5 years as repayment for a loan charged at 1% a month.

Present value	month 1	month 1	month 1	Month 1…	Month 60
	100	100	100	100	100
4495.5	100/1.01 = 99	100/1.01^2 = 98	100/1.01^2 = 97	100/1.01^2 = 96	100/1.01^{60} = 55

FV of Annuities - Formulas

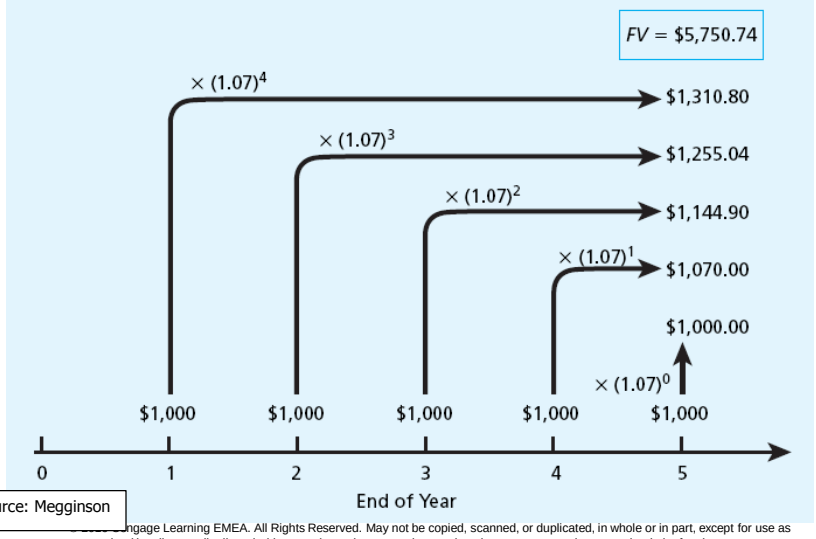
FV of an ordinary annuity:

$$FV = PMT \times \left\{ \frac{[(1 + r)^n - 1]}{r} \right\}$$

FV of an annuity due:

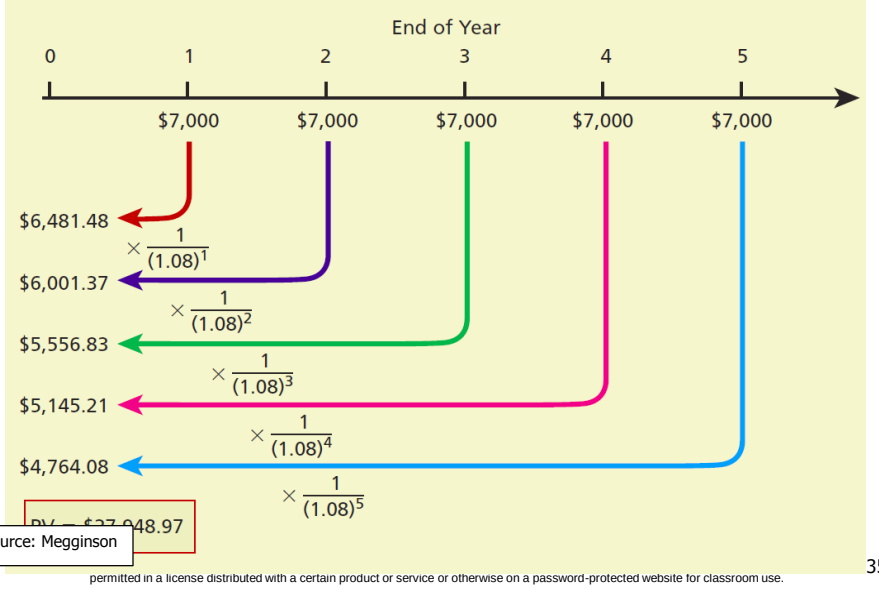
$$FV(\text{annuity due}) = PMT \times \left\{ \frac{[(1 + r)^n - 1]}{r} \right\} \times (1 + r)$$

Exercise: calculate the FV of an *ordinary annuity* of $1,000 at 7% interest annually for 5 years



Source: Megginson

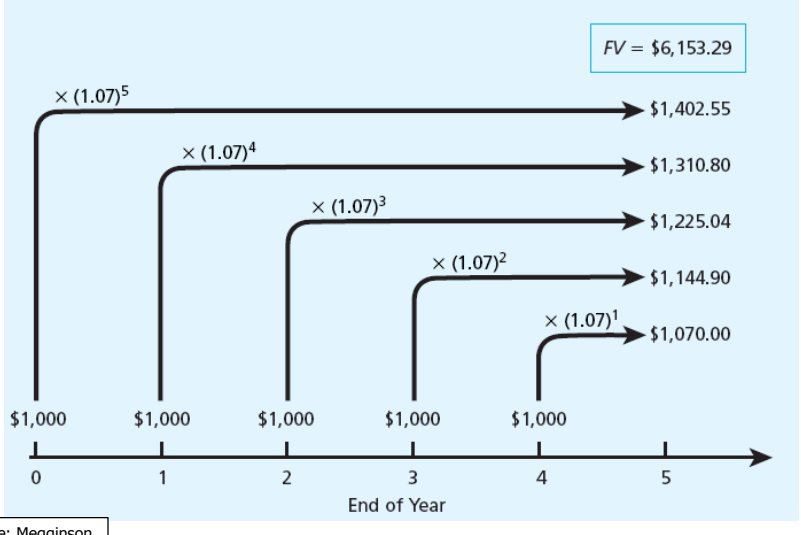
Exercise: Calculate the Present value of ordinary annuity (Payment $7000, 5 Years, 8% Interest Rate)



Source: Megginson

35

Calculate the FV of an *annuity due* of $1,000 at 7% annual interest for 5 years



Source: Megginson

Annuities the formula

$$\text{Present value of an n year annuity of €P} = P \text{ x } \left(\frac{1}{i} - \frac{1}{i \text{ x } (1+i)^n} \right)$$

The difference between a perpetuity starting now

and a perpetuity starting in n periods time

Annuities the formula

$$\text{Present value of an n year annuity of €P} = P \text{ x } \left(\frac{1}{i} - \frac{1}{i \text{ x } (1+i)^n} \right)$$

- $A_{i,n}$ = Annuity £1 for *n* periods an interest rate of *i*
- *i* = 0.01 and n = 60 $A_{\mathbf{0.01,60}}$ = (100 – 55.045) = 44.955.
- Annuity of €100 has a present value of 100 x 44.955 = €4,495.5

What's the FV of a 5-year monthly annuity of $100 at 1%/ month?

Previously calculated that the present value is €4,495.5

Therefore… €4,495.5 x 1.01^{60} = €8,166.96

Can use the formulae to move cash flows around in time

Present value of a perpetuity

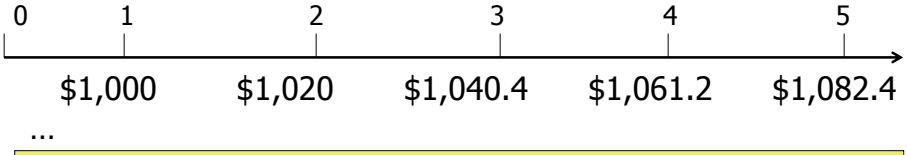
$$PV = PMT \times \frac{1}{r} = \frac{PMT}{r}$$

Present value of a growing perpetuity

$$PV = \frac{CF_1}{r - g} \qquad r > g$$

Present value of a growing perpetuity

Growing Perpetuity:
CF_1 = \$1,000; r = 7% per year; g = 2% per year



$$PV = \frac{CF_1}{r-g} = \frac{\$1{,}000}{0.07-0.02} = \$20{,}000$$

Source: Adjusted from Megginson

41

Compounding more often than annually

- Semiannually
- Quarterly
- Continuous
- The more frequently interest compounds, the greater the amount of money that accumulates.

42

Compounding Intervals

General equation where:
m = number of payments per period; and
n = number of periods

$$FV_n = PV \times \left(1 + \frac{r}{m}\right)^{m \times n}$$

- For Semiannual Compounding m = 2
- For Quarterly Compounding m = 4
- For Monthly Compounding m = 12
- For continuous Compounding m = 365 / 360

Compounding more frequently than annually
PV R125000 and interest rate 5%

Semi-annual Compounding

$$FV_2 = \text{R}125000 \times \left(1 + \frac{0.05}{2}\right)^{2 \times 2} = \text{R } 137976.61$$

Quarterly Compounding

$$FV_2 = \text{R}125000 \times \left(1 + \frac{0.05}{4}\right)^{4x2} = \text{R}138060.76$$

Nominal rate

- Stated in contracts, and quoted by banks and brokers.
- The rate is per year unless stated otherwise
- Examples:
  - 8% Quarterly = 1.08^4 -1 = 1.36 – 1 = 36% AER
  - AER = Annual Equivalent rate or EAR equivalent effective rate
  - Daily interest (365 days)
    - Simple 36% / 365 = 0.09863%
    - Actuarial $(1+x)^{365}$= 36% so $1.36^{1/365}$ -1 = x = 0.0843%

Effective annual rate

$$EAR = \left(1 + \frac{r}{m}\right)^m - 1$$

EAR_{Annual} = 12%.

EAR_Q = $(1 + 0.12/4)^4 - 1$ = 12.55%.

EAR_M = $(1 + 0.12/12)^{12} - 1$ = 12.68%.

$EAR_{D(365)}$ = $(1 + 0.12/365)^{365} - 1$ = 12.75%.

Exercise FV of PV R100 for 1 year interest 5%

To solve actuarial problems

- Find the right formula and solve for the missing value. Solve for values (how much) by equation and for *n* and *i* with a spreadsheet (trial and error)
  - Mr Y offers a ½ share in a painting to be sold in 2 years' time for an estimated €500,000. You want a 25% return how much would you offer *now* for the ½ share?
  - $250{,}000/1.25^{\mathbf{2}} = \mathbf{x}$ therefore $\mathbf{x} = €160{,}000$

To solve actuarial problems

- continued
  - You want to save €150 a month to buy a car costing €5,000. How long will you have to save if interest rates are 0.5% a month

present value of an annuity…

moved to a future point in time (*n*)

$$\left(\frac{1}{i} - \frac{1}{i \times (1+i)^n}\right) \times (1+i)^n = 5{,}000$$

  - Solution:
    - We know $i = 0.5\%$ we do not know n
    - Solve using a spreadsheet (31 months)

Spot rates

- Spot rates = invest now for a given "term" (i.e. time period) at this annual rate…

- e.g. Barclays flexible bond rates Nov 2015
  1yr 1.1%; 2yr 1.2%; 3yr 1.8%

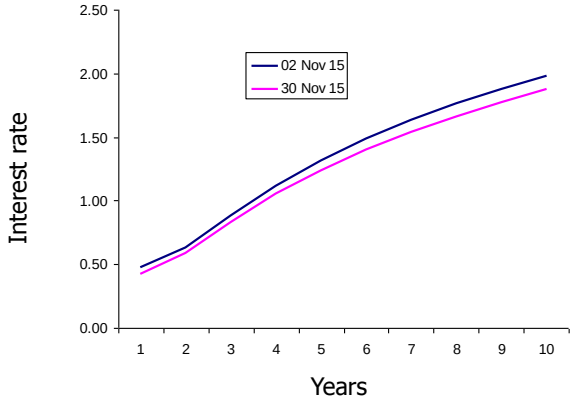
Investment for varying spot rates and their terms	End of term value
£1,000 for 1 year spot = £1,000 x 1.011 =	£1,011
£1,000 for 2 year spot = £1,000 x 1.012^2 =	£1,024
£1,000 for 3 year spot = £1,000 x 1.018^3 =	£1,055

Forward rates

- Forward rates = annual rate to invest in *x* years' time

Investment for varying *spot* rates & terms	end of term value	Implied *annual* forward rates
£1,000 for 1 year spot 1.1%	£1,011	Rate from now so only a spot rate of 1.1%
£1,000 for 2 year spot @1.2%	£1,024	1 year forward rate:1024/1011 – 1 = 1.29%
£1,000 for 3 year spot @1.8%	£1,055	2 year forward rate:1,055/1,024 – 1 = 3.01%
Check £1,000 x 1.011 x 1.0129 x 1.0301 = £1055		

Spot Yield curve on U.K. Government bonds



source: Bank of England website

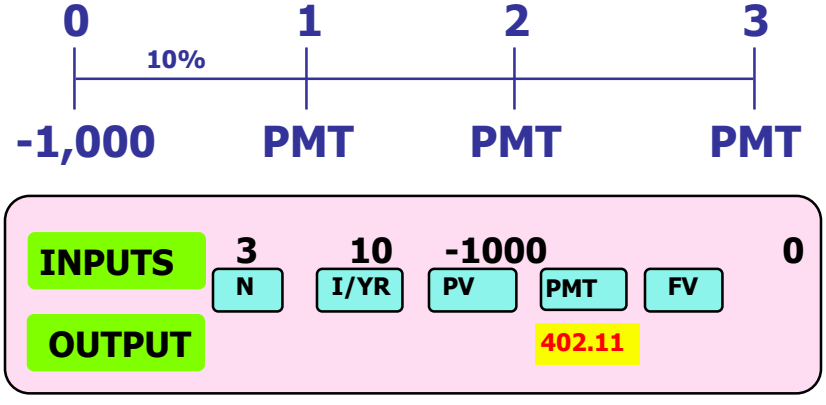
Time value of money

- Values time and risk
- The main model for borrowing and lending calculations (e.g. bonds)
- The main valuation model in finance for projects and risky investments (e.g.shares) but not the only model

Amortization

Construct an amortization schedule for a R1,000, 10% annual rate loan with 3 equal payments.

Step 1: Find the required payments.



Step 2: Find interest charge for Year 1.

$$INT_t = Beg\ bal_t\ (r)$$
$$INT_1 = R1{,}000(0.10) = \boxed{R100.}$$

Step 3: Find repayment of principal in Year 1.

$$\text{Capital payment} = PMT - INT$$
$$= R402.11 - R100$$
$$= \boxed{R302.11}$$

55

Step 4: Find ending balance after Year 1.

$$\text{End bal} = \text{Beg balance} - \text{Capital Payment}$$
$$= R1{,}000 - R302.11 = R697.89.$$

Repeat these steps for Years 2 and 3 to complete the amortization table.

56

Payment Schedule

Yr	Beg bal	Pmt	Int	Prin pmt	End bal
1	R1,000	R402	R100	R302	R698
2	698	402	70	332	366
3	366	402	37	366	0
TOT		1,206.34	206.34	1,000	

Interest declines. Tax implications.

Implied interest or growth rates

- Lump sums: the interest or growth rate of a single cash flow over time

$$(1 + r)^n = \frac{FV}{PV}$$

- Annuities: interest rate associated with an annuity

Have we achieved our goals

Are we able to:

- Do time value of money calculations
  - Future value
  - Present value
  - Rate of return / Interest rate
  - Time
  - for annuities, lump sums and unequal cash flows
- Take decisions base on time value of money

Small Case

Calculate:

The value of R1500 invested today for 5 years if the rate of return is 15% per year **= R3017.03**

PV = -R1500 FV = ? N = 5 PMT = R0 I = 15%

What would the value of R1500 be over 5 years if invested one year from today if the rate of return is 15% per year **= R2623.50**

PV = -R1500 FV = ? N = 4 PMT = R0 I = 15%

Small Case

Calculate:

The value of R1500 invested today for 5 years if the rate of return is 15% per year and compounded semi-annually **= R3091.54**

PV = -R1500 FV = ? N = 10 PMT = R0 I = 7.5%

The value of R1500 invested one year from today for 5 years if the rate of return is 15% per year and compounded semi-annually **= R2675.21**

PV = -R1500 FV = ? N = 8 PMT = R0 I = 7.5%

Small Case

Calculate:

If you want to have R15000 five years from today how much do you need to invest today if you can earn 10% per year **= R9313.81**

PV = ? FV = R15000 N = 5 PMT = R0 I = 10%

If you want to have R15000 five years from today how much do you need to invest at the end of each year if you can earn 10% per year

= R10245.20

PV = ? FV = R15000 N = 4 PMT = R0 I = 10%

Have we achieved our goals

Calculate:

If you want to have R15000 five years from today how much do you need to invest today if you can earn 10% per year compounded quarterly

PV = ? FV = R15000 N = 20 PMT = 0 I = 2.5%

If you want to have R15000 five years from today how much do you need to invest at the end of each year if you can earn 10% per year compounded quarterly

PV = ? FV = R15000 N = 16 PMT = 0 I = 2.5%

Small Case

Calculate:

If you have R15000 today and you want to have R45000 in five years what rate of return do you need to earn

PV = -R15000 FV = R45000 N = 5 PMT = 0 I = ?

If you can earn a rate of return of 15% and you have R15000 today how long will it be before you have R45000

PV = -R15000 FV = R45000 N = ? PMT = 0 I = 15%

Small Case

Construct an amortization schedule for a R2,000, 12% annual rate loan with 5 equal payments.

Year	Begin	PMT	Interest	Principle	End
1	R 2 000	R 554	R 240	R 314	R 1 686
2	R 1 686	R 554	R 202	R 352	R 1 334
3	R 1 334	R 554	R 160	R 394	R 940
4	R 940	R 554	R 113	R 441	R 499
5	R 499	R 554	R 60	R 494	R 5

Class Case Questions

Based on the financial information available calculate the following:

- Growth in sales for the past five years.
- Growth in total fix asset value over the past five years.
- Growth in total current asset value over the past five years.
- Growth in total asset value over the past five years.
- Growth in cost of sales over the past five years.
- Growth in fixed /operating cost over the past five years.
- Calculate the operating profit margin for the past five years.

Based on the above what is your assessment of the management of operations of the organization? - Compare the relative growth between related variables i.e. Sales to Asset growth

Note regarding the above to be added to this slide range on Efundi later