Brigham,Ehrhardt & Fox

Financial Management: Theory and Practice First EMEA edition

CHAPTER 6

Risk and Return

Risk and Return: Basic Concepts

- Basic return concepts (historic; expected)
- Basic risk concepts (historic; expected)
- Stand-alone risk (for an entity)
- Portfolio (market) risk
- Risk and return:
  - Individual company (CAPM/SML)
  - Portfolio (CML)

Portfolio theory: Basic Concepts

- Portfolio Theory
  - Efficient frontier
  - Capital Market Line (CML)
- Capital Asset Pricing Model (CAPM)
  - Security Market Line (SML)
  - Beta calculation
- Arbitrage pricing theory
- Fama-French 3-factor model

Chapter Outcomes

At the end of this chapter you should be:

Knowledgeable on risk management and be able to calculate:

Expected return.

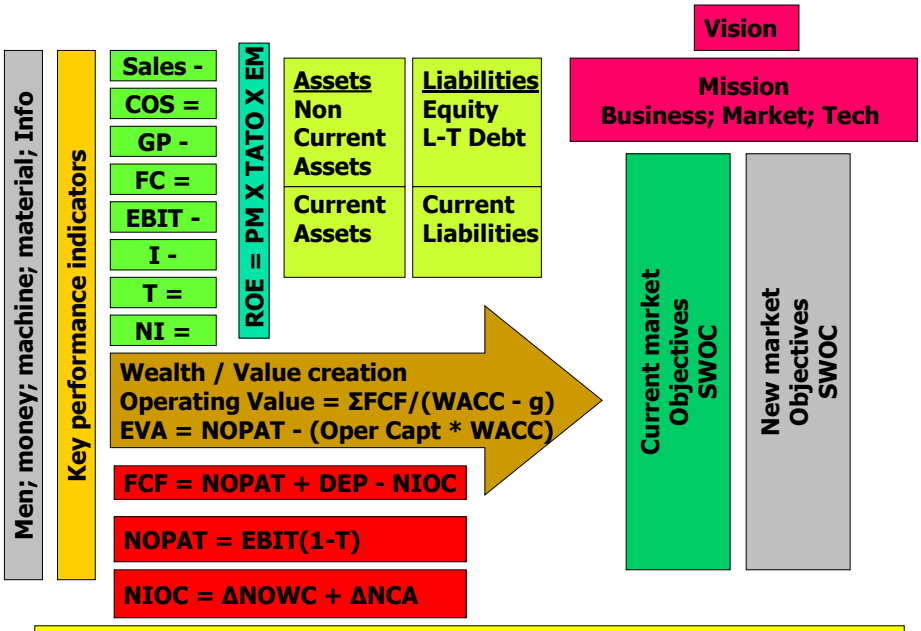
Variance.

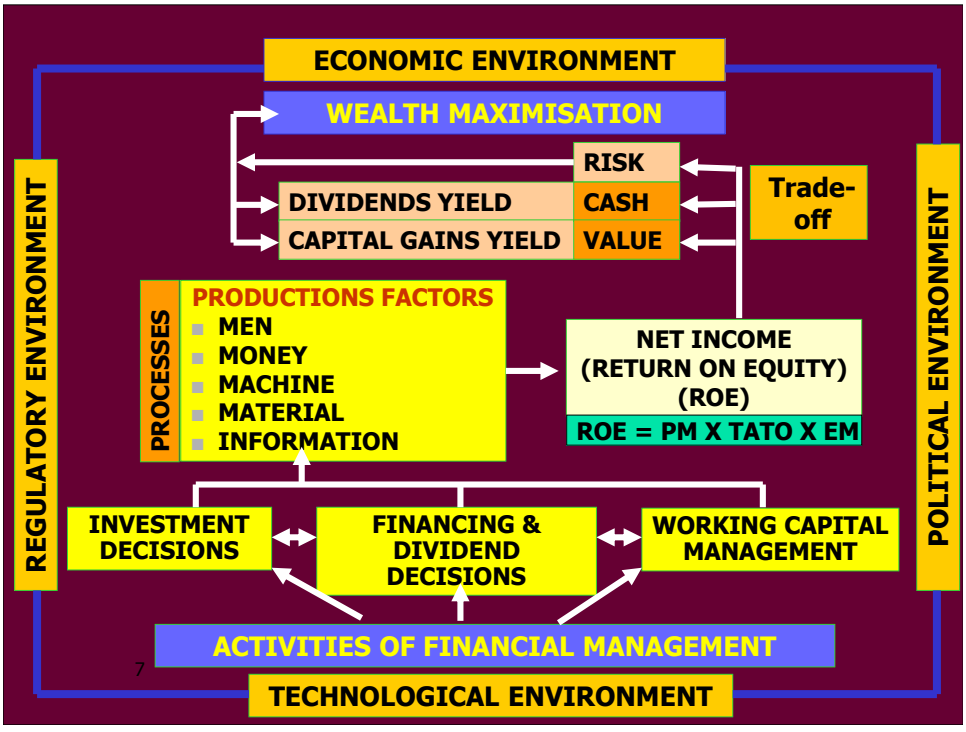
Standard deviation.

Coefficient of variation; for

Stand-alone and portfolio risk.

Discuss portfolio risk, efficient portfolios and select optimal portfolios





Risk and Return

Valuing risky assets - a task fundamental to financial management

Three-step procedure for valuing a risky asset:

1. Determine the asset's expected cash flows
2. Determine discount rate that reflects asset's risk
3. Calculate present value (PV cash inflows - PV outflows)

This is called: *Discounted cash flow (DCF)* analysis

Historical vs. Expected Returns

Decisions must be based on expected returns

Many ways to estimate expected returns and calculate risk

A simple but not necessarily correct way is to:

Assume that expected return and risk going forward will be similar to past values

What is investment risk?

- Investment returns are not known with certainty.
- **Investment risk** pertains to the probability of earning a return less than (downside risk) expected. Earning more than expected (upside risk) is not a risk it is a bonus.
- The greater the chance of a return far below the expected return, the greater the risk.

What are investment returns?

- Investment returns measure the financial results of an investment.
- Returns may be historical or prospective (anticipated).
- Returns can be expressed in:
  - Money terms.
  - Percentage terms.

An investment costs $1,000 and is sold after 1 year for $1,060.

Euro return:

€ Received - € Invested
€1,060 - €1,000 = €60.

Percentage return:

€ Return/€ Invested
€60/€1,000 = 0.06 = 6%.

Return on a single asset

Return - The total gain or loss experienced on an investment over a given period of time.

Expected sales price (New Price)

Purchase price (Old price)

Cash Flow

$$R_{t+1} = \frac{P_{t+1} - P_t + C_{t+1}}{P_t}$$

An example....

Bought for $60/share

Dividend = $6/share

Sold for $63/share

$$R = \frac{\$63 - \$60 + \$6}{\$60} = \frac{\$9}{\$60} = 15\%$$

Arithmetic Versus Geometric Returns

- ***Arithmetic*** return the simple average of annual returns: best estimate of expected return each year.

$$(R_1 + R_2 + R_3 + \ldots + R_t) / t$$

- ***Geometric*** average return the compounded annual return to an investor who bought and held a stock *t* years:

$$(1+R_1)(1+R_2)(1+R_3)\ldots.(1+R_t)]^{1/t} - 1$$

The difference between arithmetic returns and geometric returns gets bigger the more volatile the returns are

Arithmetic Versus Geometric Returns

An example....	
Year	Return
2015	-10%
2016	+13%
2017	+17%
2018	+ 8%

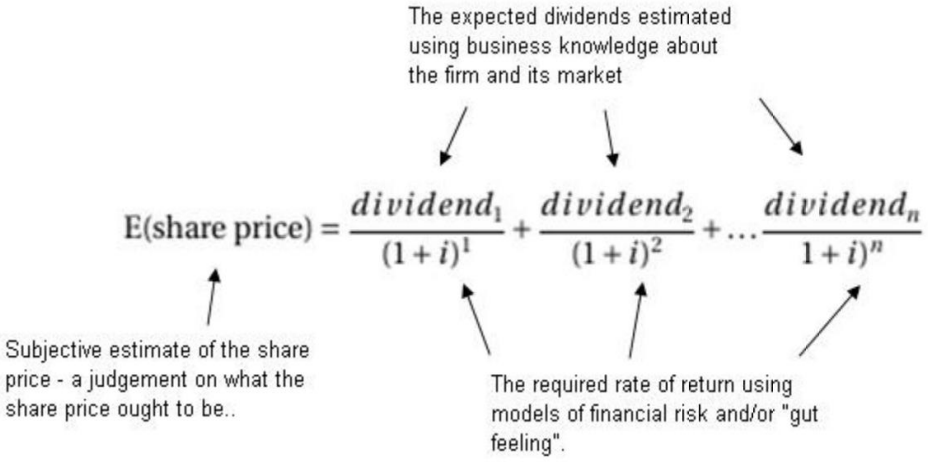
AAR = 7.00%

GAR = 6.47%

AAR =(-10 + 13 + 17 +8) / 4 = 7.00%

GAR = $[(1 + (-0.10)(1 + 0.13)(1 + 0.17)(1 + 0.08)]^{(1/4)} - 1 = 6.47\%$

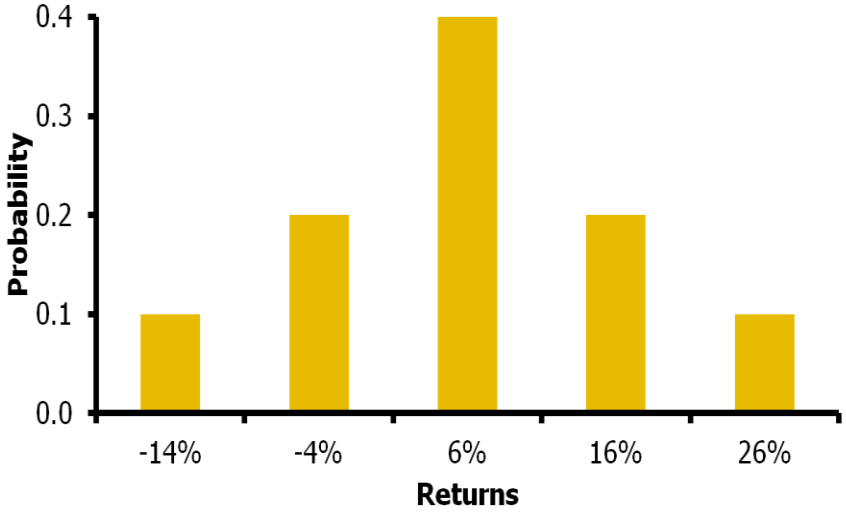
The Focus



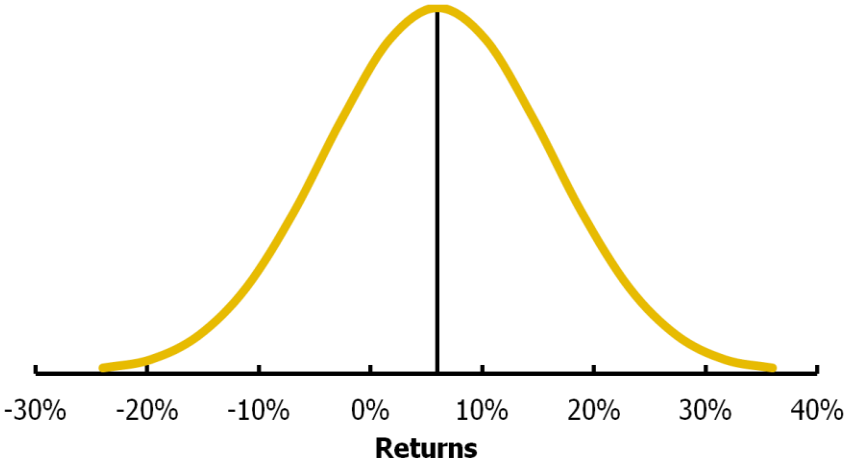
Scenarios and Returns for a share - Next Year

Scenario	Probability	Return
Worst Case	0.10	−14%
Poor Case	0.20	−4%
Most Likely	0.40	6%
Good Case	0.20	16%
Best Case	0.10	26%
	1.00	

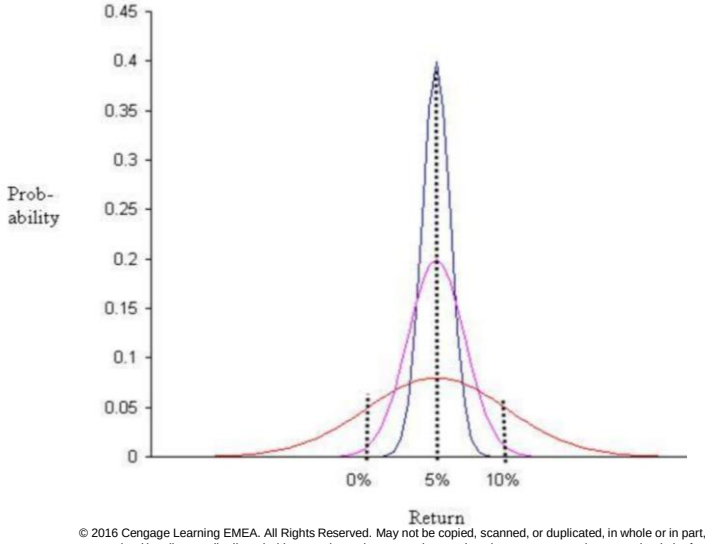
Discrete Probability Distribution for Scenarios



Continuous Probability Distribution



Returns for differing risks



Expected return

$\hat{r}$ = **expected rate of return.**

$$\hat{r} = \sum_{I=1}^{n} R_i P_i$$

$\hat{r}$ = 0.10(-14%) + 0.20(-4%)

+ 0.40(6%) + 0.20(16%)

+ 0.10(26%) = 6%.

Stand-Alone Risk: Standard Deviation

- Stand-alone risk is the risk of each asset held by itself.
- Standard deviation measures the dispersion of possible outcomes.
- For a single asset:

  Stand-alone risk = Standard deviation

Individual asset - Standard deviation of returns

$$\sigma^2 = \sum_{i=1}^{n} \left(r_i - \hat{r} \right)^2 P_i$$

$$\sigma = \sqrt{\sum_{i=1}^{n} \left(r_i - \hat{r} \right)^2 P_i} \ .$$

Standard Deviation of the Share's Return During the Next Year

$$\begin{aligned}\sigma^2 = \ & 0.10\,(-0.14 - 0.06)^2 \\ & + 0.20\,(-0.04 - 0.06)^2 \\ & + 0.40\,(\ 0.06 - 0.06)^2 \\ & + 0.20\,(\ 0.16 - 0.06)^2 \\ & + 0.10\,(\ 0.26 - 0.06)^2\end{aligned}$$

$$\sigma^2 = 0.0120$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{.0120}$$

$$\sigma = 0.1095 = 10.95\%$$

Expected return and expected risk
Case investment alternatives (Brigham & Erhardt)

Economy	Prob.	T-Bill %	Alta %	Repo %	Am F. %	Market %
Recession	0.10	8.0	-22.0	28.0	10.0	-13.0
Below avg.	0.20	8.0	-2.0	14.7	-10.0	1.0
Average	0.40	8.0	20.0	0.0	7.0	15.0
Above avg.	0.20	8.0	35.0	-10.0	45.0	29.0
Boom	0.10	8.0	50.0	-20.0	30.0	43.0
	1.00					

What is unique about the investment opportunities?

- The T-bill will return 8% regardless of the state of the economy.
- Is the T-bill riskless? Explain.
- Alta moves with the economy, so it is positively correlated with the economy. This is the typical situation.
- Repo moves counter to the economy. Such negative correlation is unusual.

Expected rate of return of an individual asset.

$\hat{r}$ = **expected rate of return.**

$$\hat{r} = \sum_{I=1}^{n} R_i P_i$$

Alta - case

$\hat{r}_{Alta}$ = 0.10(-22%) + 0.20(-2%) + 0.40(20%)

+ 0.20(35%) + 0.10(50%) = 17.4%.

Expected rate of return individual assets.

	$\hat{r}$
Alta	17.4%
Market	15.0
Am. F	13.8
T-bill	8.0
Repo	1.7

- Alta has the highest rate of return.
- Does that make it best?

Risk interpreted.

- Standard deviation measures the stand-alone risk of an investment.
- The larger the standard deviation, the higher the probability that returns will be far below / above the expected return.
- Coefficient of variation is an alternative measure of stand-alone risk.

Expected Return (E(R)) and Risk Choice)

Security	Expected return	Risk, σ
Alta Inds.	17.4%	20.0%
Market	15.0	15.3
Am. Foam	13.8	18.8
T-bills	8.0	0.0
Repo Men	1.7	13.4

Coefficient of Variation (CV) = Standard deviation / Expected return

$CV_{T\text{-}BILLS}$	= 00.0% / 8.0%	=	0.0
$CV_{Alta\ Inds}$	= 20.0% / 17.4%	=	1.1
$CV_{Repo\ Men}$	= 13.4% / 1.7%	=	7.9
$CV_{Am.\ Foam}$	= 18.8% / 13.8%	=	1.4
CV_{M}	= 15.3% / 15.0%	=	1.0

CV = Units of standard deviation / unit of return

E(R) and CV (Choice ?)

Security	Expected return	Risk: σ	Risk: CV
Alta Inds	17.4%	20.0%	1.1
Market	15.0	15.3	1.0
Am. Foam	13.8	18.8	1.4
T-bills	8.0	0.0	0.0
Repo Men	1.7	13.4	7.9

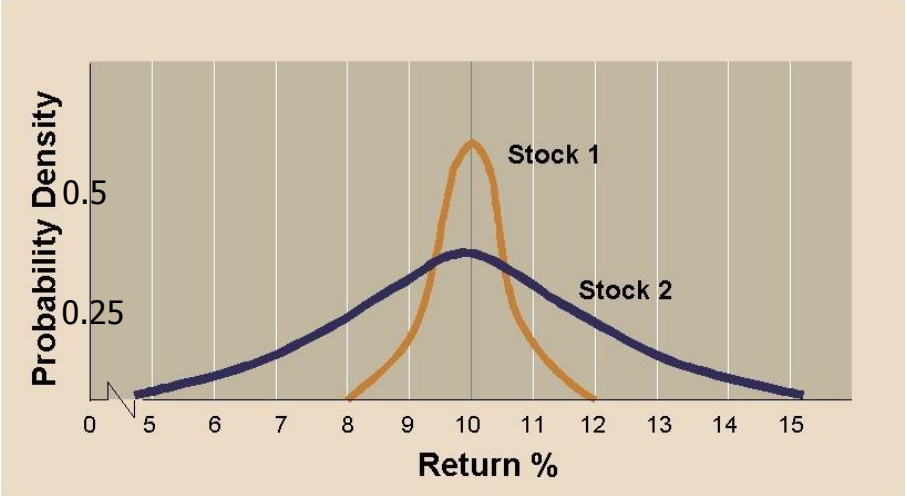
Expected return for a *Portfolio*

- Most investors hold multiple asset portfolios
- Key insight of *portfolio theory*: asset return adds linearly, but risk is (almost always) reduced in a portfolio

$$E(R_p) = w_1 * E(R_1) + w_2 * E(R_2) + w_3 * E(R_3) + w_N * E(R_N)$$

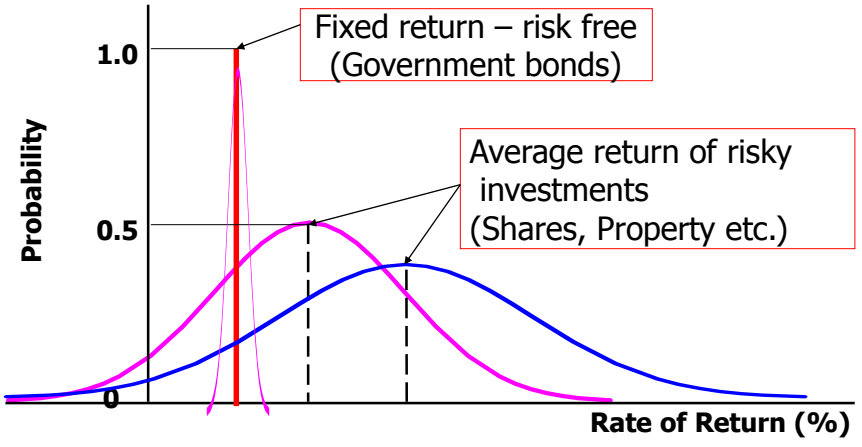
The expected return of portfolio is the weighted average of expected returns of the stocks

Two assets same expected return (10%) but different distributions

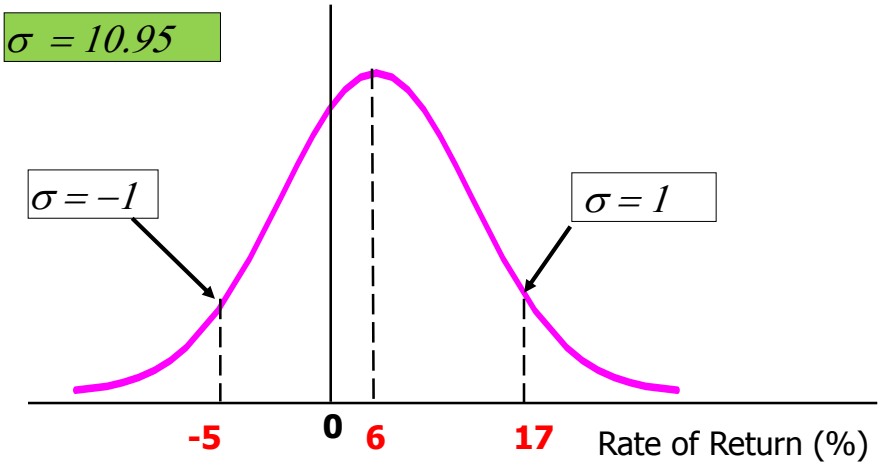


Probability Distribution - Earnings volatility .

Probability distributions tell what outcomes are possible and how likely each outcome is.



Normal distributions



Understanding the Standard Deviation

- If the returns are normally distributed:
  - Outcome will be more than 1 σ away from $\hat{r}$ about 31.74% ≈ 32% of the time:
    - 16% of the time below $\hat{r}-\sigma$
    - 16% of the time above $\hat{r}+\sigma$.

    } 32%
  - If $\hat{r}$ = 6% and σ =10.95% ≈ 11%:
    - 16% of the time return <−5% = 6% − 11%
    - 16% of the time return > 17% = 6% + 11

Useful in Comparing Investments

- Investments with bigger standard deviations have more risk.
- High risk doesn't mean you should reject the investment, but:
  - You should know the risk before investing
  - You should expect a higher return as compensation for bearing the risk.

Using Historical Data to Estimate Risk

- Analysts often use discrete outcomes to analyze risk for projects
- But for investments, most analysts normally use historical data rather than discrete forecasts to estimate an investment's risk unless it is a very special situation.
- Most analysts use:
  - 48 to 60 months of monthly data, or
  - 52 weeks of weekly data, or
  - Shorter period using daily data.
- Use annual returns here for sake of simplicity.

Formulas for a Sample of *t* Historical Returns

- $\bar{r}_{Avg} = \frac{\sum_{t=1}^{T} \bar{r}_t}{T}$

$$\text{Est.}\,\sigma = S = \sqrt{\frac{\sum_{t=1}^{T} \left(\bar{r}_t - \bar{r}_{Avg}\right)^2}{T-1}}$$

In Excel for example "=stdev(A1 : A100)"

Risk of a Single Asset

Example: Year 2015 2016 2017 2018
Return -10% +13% +17% +8%

Average annual return = 7%

Variance :

$$\frac{(-10\%-7\%)^2 + (13\%-7\%)^2 + (17-\%7\%)^2 + (8\%-7\%)^2}{4-1}$$

$= (-17^2 + 6^2 + 10^2 + 1^2) = 426 / 3 = 142$ (variance)

Standard deviation $= \sqrt{142}$

$= 11.91$

Historical Data for Stock Returns

Year	Market	Blandy	Gourmange
1	30%	26%	47%
2	7	15	−54
3	18	−14	15
4	−22	−15	7
5	−14	2	−28
6	10	−18	40
7	26	42	17
8	−10	30	−23
9	−3	−32	−4
10	38	28	75

Standard Deviations for Stand-Alone Investments

- Use formulas shown previously (tedious) or use *Excel* (easy)
- What is Blandy's stand-alone risk?
- Note: analysts often use past risk as a predictor of future risk, but **past returns are often not a good prediction of future returns**.

	Market	Blandy	Gourmange
Average return	**8.0%**	**6.4%**	**9.2%**
Standard deviation	**20.1%**	**25.2%**	**38.6%**

How risky is the share in Blandy?

Assumptions:

- Returns are normally distributed, so about 16% of the time, return will be less than the average minus σ; about 16% of the time the return will be greater than the average plus σ.
- σ is 25.2%
- Expected return is about 6.4%.

About 16% of the time, return will be:

- < −18.8% (6.4%−25.2% = −18.8%)
- > 31.6% (6.4%+25.2% = 31.6%)
- Stocks are very risky!

Expected return for a ***Portfolio***

- Most investors hold multiple asset portfolios
- Key insight of *portfolio theory*: asset return adds linearly, but risk is (almost always) reduced in a portfolio

$$E(R_p) = w_1 * E(R_1) + w_2 * E(R_2) + w_3 * E(R_3) + w_N * E(R_N)$$

The expected return of portfolio is the weighted average of expected returns of the stocks

Example: 2-Stock Portfolio

Invest 75% in Blandy and 25% in Gourmange

- Form a portfolio by selling 25% of the Blandy stock and investing it in the higher-risk Gourmange stock.
- The portfolio return each year will be:
  - $\bar{r}_{P,t} = w_{Blandy}(\bar{r}_{Blandy,t}) + w_{Gour.}(\bar{r}_{Gour.,t})$
  - $\bar{r}_{P,t} = 0.75(\bar{r}_{Blandy,t}) + 0.25(\bar{r}_{Gour.,t})$

Stocks and Portfolio Returns

Year	Blandy	Gourmange	Portfolio
1	26%	47%	31.3%
2	15	−54	−2.3
3	−14	15	−6.8
4	−15	7	−9.5
5	2	−28	−5.5
6	−18	40	−3.5
7	42	17	35.8
8	30	−23	16.8
9	−32	−4	−25.0
10	28	75	39.8

Two-Asset Portfolio Standard Deviation

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2$$

$$\text{Standard Deviation} = \sqrt{\sigma_p^2}$$

Correlation between stocks influences portfolio volatility

Historical values

- The portfolio's average return is the weighted average of the Shares/Stocks' average returns.
- BUT The portfolio's standard deviation is *less* than either stock's σ!
- What explains this?

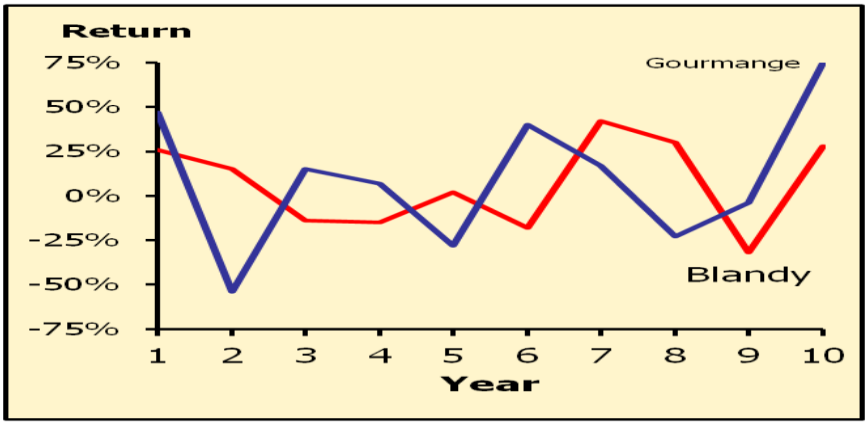
	Blandy	Gourmange	Portfolio
Average return	6.4%	9.2%	7.1%
Standard deviation	25.2%	38.6%	**22.2%**

The Importance of Covariance

- Risk reduction is achieved in portfolios because fluctuations in **return** of one asset partially off set fluctuations in the other
- Risk of a portfolio depends crucially on whether the returns on the portfolio's components move *together* or *in opposite directions*
- **Covariance:** Statistical measurement of the co-movements of two random variables

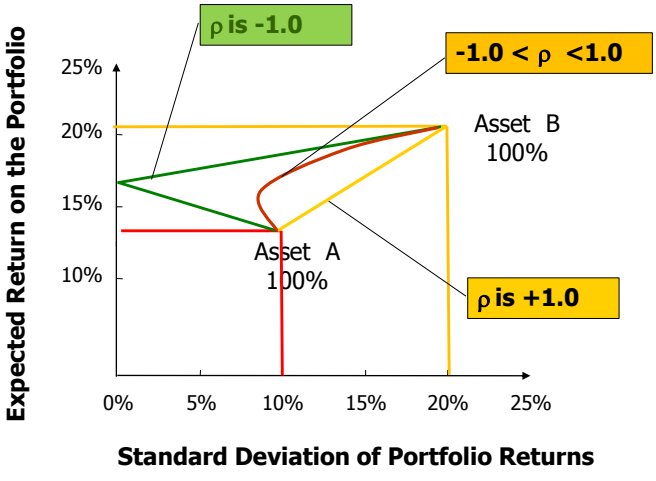
$$\mathrm{Cov}(R_1, R_2) = \sigma_{12} = \frac{\left[\sum_{t=1}^{N} \left(R_{1t} - \overline{R}_1\right)\left(R_{2t} - \overline{R}_2\right)\right]}{N-1}$$

How closely do the returns follow one another?



Notice that the returns don't move in perfect unison: Sometimes one is up and the other is down.

Correlation Coefficients And Risk Reduction For Two-Asset Portfolios



In a 2-Share Portfolio

- **r = −1**
  - 2 stocks can be combined to form a riskless portfolio: $\sigma_p = 0$.
- **r = +1**
  - Risk is not "reduced"
  - σ_p is just the weighted average of the 2 stocks' standard deviations.
- **−1 < r < −1**
  - Risk is reduced but not eliminated.

Variance – Covariance Matrix

Asset	1	2	3	4	5
1	$\left(\frac{1}{5}\right)^2 \sigma_1^2$	$\left(\frac{1}{5}\right)^2 \sigma_{12}$	$\left(\frac{1}{5}\right)^2 \sigma_{13}$	$\left(\frac{1}{5}\right)^2 \sigma_{14}$	$\left(\frac{1}{5}\right)^2 \sigma_{15}$
2	$\left(\frac{1}{5}\right)^2 \sigma_{21}$	$\left(\frac{1}{5}\right)^2 \sigma_2^2$	$\left(\frac{1}{5}\right)^2 \sigma_{23}$	$\left(\frac{1}{5}\right)^2 \sigma_{24}$	$\left(\frac{1}{5}\right)^2 \sigma_{25}$
3	$\left(\frac{1}{5}\right)^2 \sigma_{31}$	$\left(\frac{1}{5}\right)^2 \sigma_{32}$	$\left(\frac{1}{5}\right)^2 \sigma_3^2$	$\left(\frac{1}{5}\right)^2 \sigma_{34}$	$\left(\frac{1}{5}\right)^2 \sigma_{35}$
4	$\left(\frac{1}{5}\right)^2 \sigma_{41}$	$\left(\frac{1}{5}\right)^2 \sigma_{42}$	$\left(\frac{1}{5}\right)^2 \sigma_{43}$	$\left(\frac{1}{5}\right)^2 \sigma_4^2$	$\left(\frac{1}{5}\right)^2 \sigma_{45}$
5	$\left(\frac{1}{5}\right)^2 \sigma_{51}$	$\left(\frac{1}{5}\right)^2 \sigma_{52}$	$\left(\frac{1}{5}\right)^2 \sigma_{53}$	$\left(\frac{1}{5}\right)^2 \sigma_{54}$	$\left(\frac{1}{5}\right)^2 \sigma_5^2$

The variance of individual assets account only for 1/25th of the portfolio variance

The covariance terms determine to a large extent the variance of the portfolio

Correlation Coefficient ($\rho_{i,j}$)

- Loosely speaking, the correlation (ρ) coefficient measures the tendency of two variables to move together.
- Estimating $\rho_{i,j}$ with historical data is tedious:

$$\frac{\sum_{t=1}^{T}\left(\bar{r}_{i,t}-\bar{r}_{i,Avg}\right)\left(\bar{r}_{j,t}-\bar{r}_{j,Avg}\right)}{\sqrt{\left[\sum_{t=1}^{T}\left(\bar{r}_{i,t}-\bar{r}_{i,Avg}\right)^{2}\right]\left[\sum_{t=1}^{T}\left(\bar{r}_{j,t}-\bar{r}_{j,Avg}\right)^{2}\right]}}$$

Portfolio risk and return under uncertainty

Assume a two-stock portfolio with R50,000 in Alta (50%) and R50,000 in Repo (50%).

Calculate $\hat{r}_p$ and σ_p

Portfolio assessment under uncertainty

		Estimated Return (%) 50% Alta 50% Repo		
Economy	Prob.	Alta	Repo	Port.
Recession	0.10	-22.0	28.0	3.0
Below avg.	0.20	-2.0	14.7	6.4
Average	0.40	20.0	0.0	10.0
Above avg.	0.20	35.0	-10.0	12.5
Boom	0.10	50.0	-20.0	15.0

$\hat{r}_p$ = (3.0%) 0.10 + (6.4%) 0.20 + (10.0%) 0.40 + (12.5%) 0.20 + (15.0%) 0.10 = 9.6%.

Portfolio Return

$\hat{r}_{Alta}$ = 0.1 x -22%) + 0.2 x -2 + 0.4 x 20 + 0.2 x 35+ 0.1 x 50= 17.4%.

$\hat{r}_{Repo}$ =

r_p is a weighted average:

$$\hat{r}_p = \sum_{i=1}^{n} w_i \hat{r}_i.$$

$\hat{r}_p$ = 0.5 (17.4%) + 0.5 (1.7%) = 9.6%.

$\hat{r}_p$ is between $\hat{r}_{Alta}$ and $\hat{r}_{Repo}$.

Portfolio standard deviation

- $\sigma_p = ((3.0 - 9.6)^2\ 0.10 + (6.4 - 9.6)^2\ 0.20 + (10.0 - 9.6)^2\ 0.40 + (12.5 - 9.6)^2\ 0.20 + (15.0 - 9.6)^2\ 0.10)^{1/2} = 3.3\%$

- The portfolio provides average return but much lower risk. The key here is negative correlation.

NB – σ_p **is not** **equal** to the weighted avg of the standard deviations of the assets in the portfolio

(0.5 *20% plus 0.5*13.4% = 16.7)

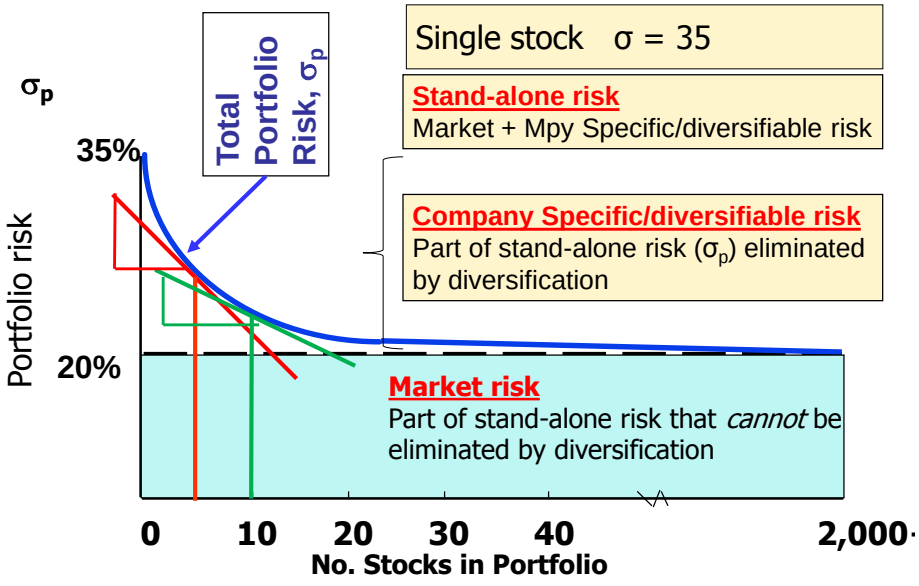
How is the variance of portfolio influenced by number of assets in portfolio?

Adding Shares to a Portfolio

- What would happen to the risk of an average 1-stock portfolio as more randomly selected stocks were added?

- σ_p would decrease because the added stocks would not be perfectly correlated.

Effect of diversification on portfolio variance



61

Stand-alone risk = Market risk + Diversifiable risk

- Market risk is that part of a security's stand-alone risk that cannot be eliminated by diversification.
- Firm-specific, or diversifiable, risk is that part of a security's stand-alone risk that can be eliminated by diversification.

62

Conclusions

- As more shares are added, each new share has a smaller risk-reducing impact on the portfolio.
- s_p falls very slowly after about 40 stocks are included. The lower limit for s_p is about 20% = s_M .
- By forming well-diversified portfolios, investors can eliminate about half the risk of owning a single stock.

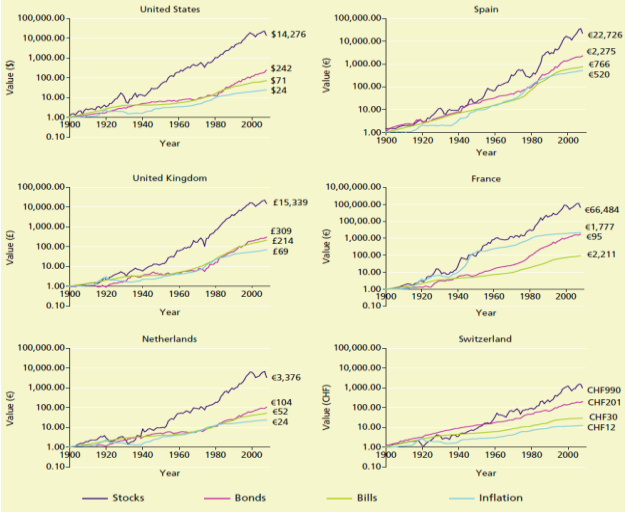
Can an investor holding one stock earn a return commensurate with its risk?

- No. Rational investors will minimize risk by holding portfolios.
- Investors bear only market risk, so prices and returns reflect the amount of market risk an individual stock brings to a portfolio, *not the stand-alone risk of individual stock*.

Market Risk Due to an Individual Stock

- How do you measure the amount of market risk that an individual stock brings to a well-diversified portfolio?
- William Sharpe developed the Capital Asset Pricing Model (CAPM) to answer this question.

Return of alternative investments in different countries, 1900-2008



Returns - Bills, Bonds, Stocks *1900 - 2003*

	Nominal (%)			Real (%)		
Asset Class	Average	Best Year	Worst Year	Average	Best Year	Worst Year
Bills	4.1	14.7	0.0	1.1	19.7	-15.1
Bonds	5.2	40.4	-9.2	2.3	35.1	-19.4
Stocks	11.7	57.6	-43.9	8.5	56.8	-38

Risk premium = $(r - r_f)$

Stocks and bills = 7.6% (nominal 11.7 – 4.1)
Stocks and bonds = 6.5% (nominal 11.7 – 5.2)

Stocks and bills = 7.4% (real 8.5 – 1.1)
Stocks and bonds = 6.2% (real 8.5 – 2.3)

Risk and Return Fundamentals

Equity risk premium: the difference between equity returns (r) and returns on safe investments(r_f)

Where : **Premium = $(r - r_f)$**

Implies that stocks are riskier than bonds or bills

Trade-off always arises between expected risk and expected return

Volatility of stocks relative to bonds / bills depends on the time horizon over which investment returns are measured

Systematic Risk and Asset Pricing

- Investors can only expect compensation for **systematic risk**:
  - Contribution of an asset's risk to a diversified portfolio is measured by beta

- The **capital asset pricing model (CAPM)** relates an asset's return to its systematic risk.
  - Assumes that rational, risk-averse investors select efficient portfolios

Beta measures the change in an individual assets' return against the change in total market return

How is market / systemetic risk measured for individual securities?

- Market risk, which is relevant for stocks held in well-diversified portfolios, is defined as the contribution of a security to the overall riskiness of the portfolio.

- It is measured by a stock's beta coefficient. For stock i, its beta is:

$$b_i = (r_{iM}\, s_i) / s_M$$

In addition to measuring a stock's contribution of risk to a portfolio, beta also measures the stock's volatility relative to the market.

The Capital Asset Pricing Model (CAPM)

CAPM model for firm *i* $E(r_i) = r_f + [E(r_m) - r_f]\frac{\sigma_{im}}{\sigma_m^2}$

normally written as:

$$E(r_i) = r_f + \beta_i(E(r_m) - r_f)$$

where:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \quad OR \quad \frac{COV(r_i, r_m)}{VAR(r_m)}$$

Market Risk and CAPM

Expected return for firm *i* the discount rate applied to value of the firm one year from now

$$E(r_i) = r_f + \beta_i(E(r_m) - r_f)$$

risk free return

a portion of the market risk premium

Market risk premium

in expanded form $E(r_i) = r_f + [E(r_m) - r_f]\frac{\sigma_{im}}{\sigma_m^2}$

note that $\frac{[E(r_m) - r_f]}{\sigma_m^2}$ is constant only σ_{im}

or **Cov(r_i,r_m)** defines the required return of a share… as in the variance covariance matrix earlier.

Capital Asset Pricing Model (CAPM)

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

Only beta changes from one security to the next. For that reason, analysts classify the CAPM as a single-factor model, meaning that just one variable explains differences in returns across securities.

Market Risk and Beta

$$E(r_i) = r_f + \beta_i(E(r_m) - r_f)$$

- $\beta_i = 1$ The return required on share i is the same as the market portfolio.
- $\beta_i > 1$ The return required on share i is greater than the market portfolio, the share is riskier than the market.
- $\beta_i < 1$ The return required on share i is less than that of the market portfolio, the share is less risky than the market portfolio.

- Most stocks have betas in the range of 0.5 to 1.5.
- Can a stock have a negative beta?

What are the CAPM assumptions?

- Investors think in terms of a single holding period.
- All investors have identical expectations.
- Investors can borrow / lend money unlimited at the risk-free rate.
- All assets are perfectly divisible.
- There are no taxes and no transactions costs.
- All investors are price takers, that is, investors' buying and selling won't influence stock prices.
- Quantities of all assets are given and fixed.

Can an investor holding one stock earn a return commensurate with its risk?

- No. Rational investors will minimize risk by holding portfolios.
- They bear only market risk, so prices and returns reflect this lower risk.
- The one-stock investor bears higher (stand-alone) risk, so the return is less than that required by the risk.

Using a regression to estimate beta

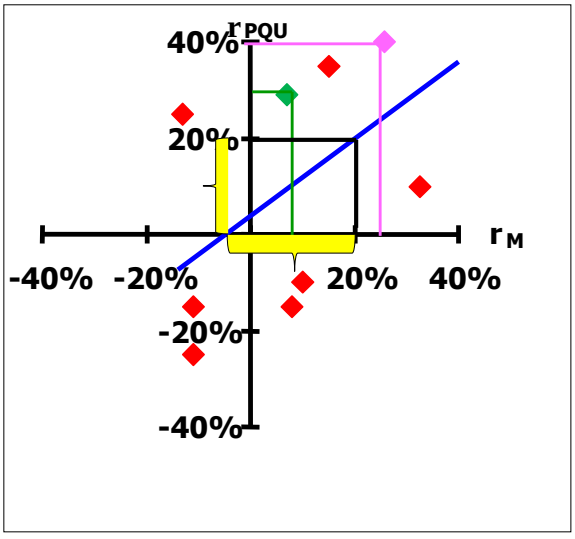
- Run a regression with returns on the stock in question plotted on the Y axis and returns on the market portfolio plotted on the X axis.

- The slope of the regression line, which measures relative volatility, is defined as the stock's beta coefficient, or b.

Use the historical stock returns to calculate the beta for PQU. (Slides Brigham & Erhardt)

Period	Market	PQU
1	**25.7%**	**40.0%**
2	8.0%	-15.0%
3	-11.0%	-15.0%
4	15.0%	35.0%
5	32.5%	10.0%
6	**13.7%**	**30.0%**
7	40.0%	42.0%
8	10.0%	-10.0%
9	-10.8%	-25.0%
10	-13.1%	25.0%

Calculating Beta for PQU



What is beta for PQU?

The regression line, and hence beta, can be found using a calculator with a regression function or a spreadsheet program. In this example, b = 0.83.

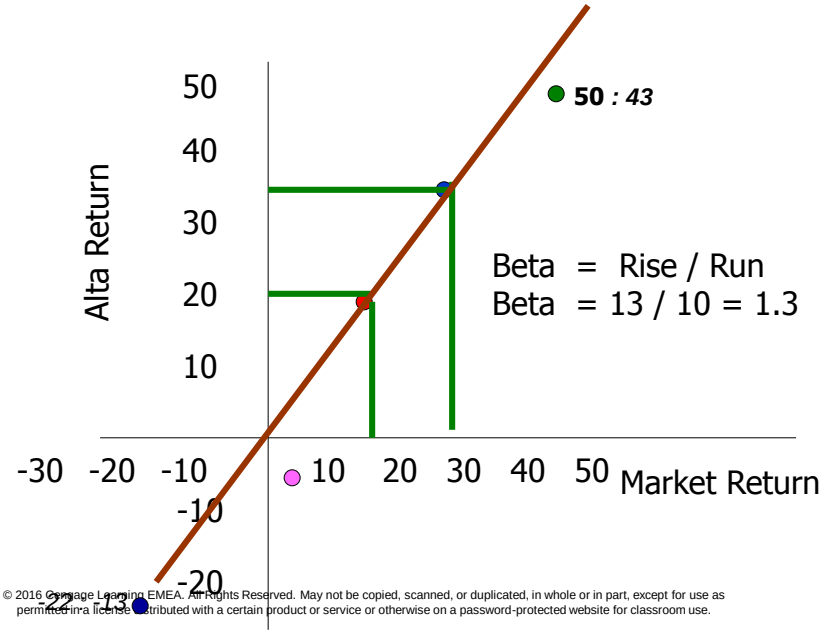
Calculating beta in practice

- In South Africa analysts use the JSE All Share Index (J 203) to find the market return.
- Analysts typically use four or five years' of monthly returns to establish the regression line.
- Some analysts use 52 weeks of weekly returns.

Expected return and expected risk
Investment alternatives (Brigham & Erhardt)

Economy	Prob.	T-Bill	Alta	Repo	Am F.	MP
Recession	0.10	8.0%	-22.0%	28.0%	0.0%	-13.0%
Below avg.	0.20	8.0	-2.0	14.7	-10.0	1.0
Average	0.40	8.0	20.0	0.0	7.0	15.0
Above avg.	0.20	8.0	35.0	-10.0	45.0	29.0
Boom	0.10	8.0	50.0	-20.0	30.0	43.0
	1.00					

Altas' expected beta



Expected Return versus Market Risk

Security	Expected return	Risk, b
Alta	17.4%	1.29
Market	15.0	1.00
Am. Foam	13.8	0.68
T-bills	8.0	0.00
Repo Men	1.7	-0.86

Which of the alternatives is best?

Use the SML to calculate each alternative's required return.

- The Security Market Line (SML) is part of the Capital Asset Pricing Model (CAPM).
- SML: $r_i = r_F + (RP_M)b_i$.
- Assume r_F = 8%; r_M = r_M = 15%.
- $RP_M = (r_M - r_F) = 15\% - 8\% = 7\%$.

RR (1-T)

Required Rates of Return

$$r_{Alta} = 8.0\% + (7\%)(1.30)$$
$$= 8.0\% + 9.0\% = \mathbf{17.0\%.}$$

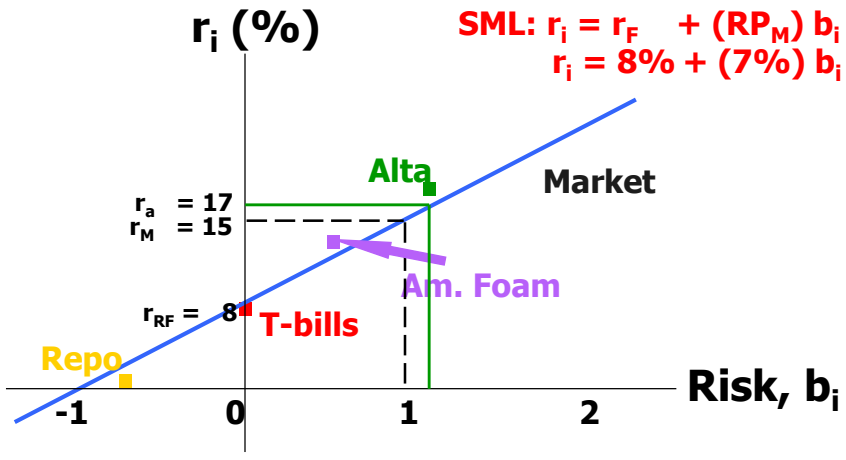
$$r_M = 8.0\% + (7\%)(1.00) = \mathbf{15.0\%.}$$
$$r_{Am.\ F.} = 8.0\% + (7\%)(0.68) = \mathbf{12.8\%.}$$
$$r_{T\text{-}bill} = 8.0\% + (7\%)(0.00) = \mathbf{8.0\%.}$$
$$r_{Repo} = 8.0\% + (7\%)(-0.86) = \mathbf{2.0\%.}$$

Expected versus Required Returns

	$\hat{r}$	r	
Alta	17.4%	17.0%	Undervalued
Market	15.0	15.0	Fairly valued
Am. F.	13.8	12.8	Undervalued
T-bills	8.0	8.0	Fairly valued
Repo	1.7	2.0	Overvalued

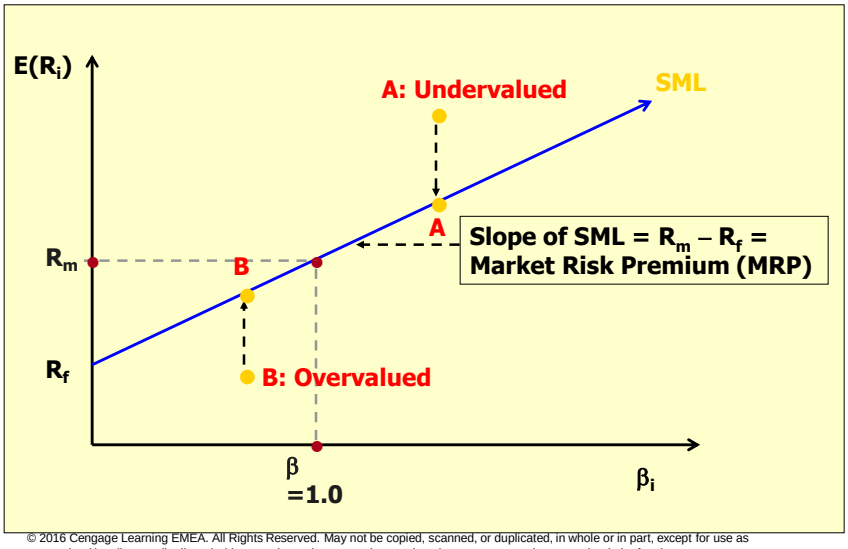
Over/Under valued

Constructing the SML

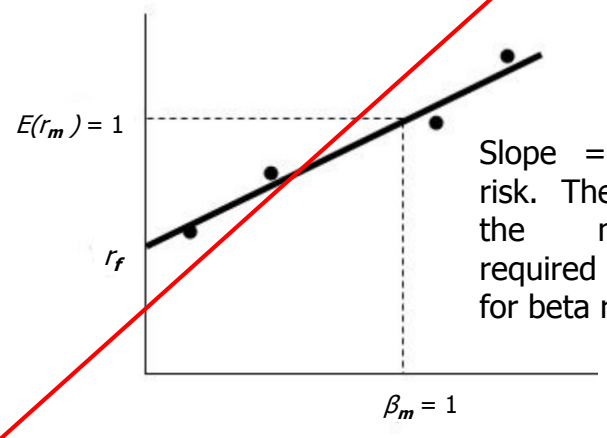


SML and Investment Alternatives

Figure 6.8 The Security Market Line



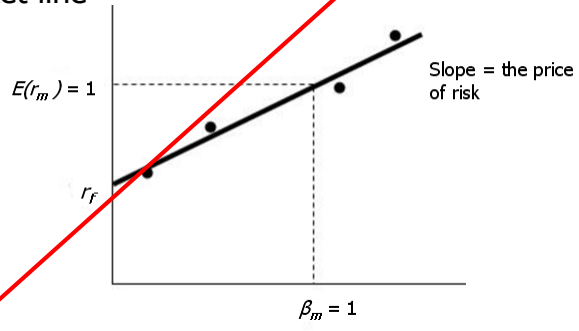
The Security Market Line: Relating Risk and Required Return



Slope = the price of risk. The steeper it is the more return required to compensate for beta risk

Risk and Return in the Stock Market

- We assume that you cannot consistently get a higher return for a given risk than that dictated by the security market line



Beta for a portfolio of:50% Alta; 50% Repo

b_p = Weighted average

= $0.5(b_{Alta}) + 0.5(b_{Repo})$

= $0.5(1.29) + 0.5(-0.86)$

= 0.22.

Required return of the Alta / Repo portfolio?

r_p = Weighted average r

= 0.5(17%) + 0.5(2%) = 9.5%.

Or use SML:

$r_p = r_F + (RP_M)\, b_p$

= 8.0% + 7%(0.22) = 9.5%.

The Security Market Line

- Plots the relationship between expected return and betas
- In equilibrium, all assets lie on this line
  - If stock lies above the line
    - Expected return is too high
    - Investors bid up price until expected return falls
  - If stock lies below the line
    - Expected return is too low
    - Investors sell stock, driving down price until expected return rises

Required Return for Blandy

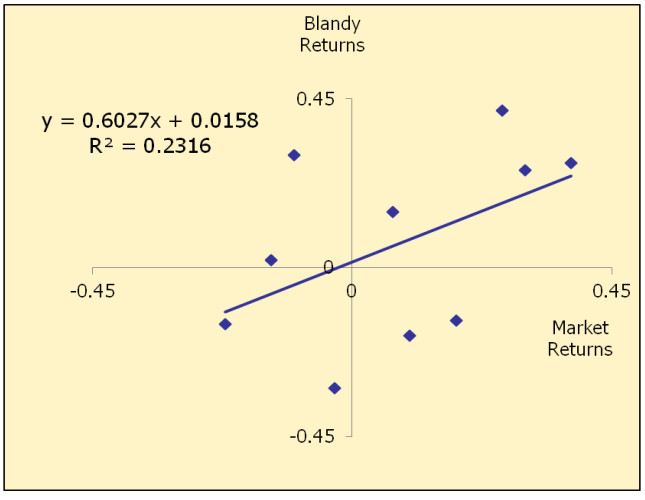
- Inputs:
  - r_{RF} = 4% (given)
  - $E(r_m - r_f)$ = 5% (given)
  - β_i = 0.60 (estimated)
- $r_i = r_{RF} + b_i (E(r_m - r_f))$

$$\mathbf{r_i = 4\% + 0.60(5\%) = 7\%}$$

Using a Regression to Estimate Beta

- Run a regression with returns on the stock plotted on the Y-axis and returns on the market portfolio plotted on the X-axis.
- The slope of the regression line is equal to the stock's beta coefficient.

Excel: Plot Trendline Right on Chart



y = Blandy's returns

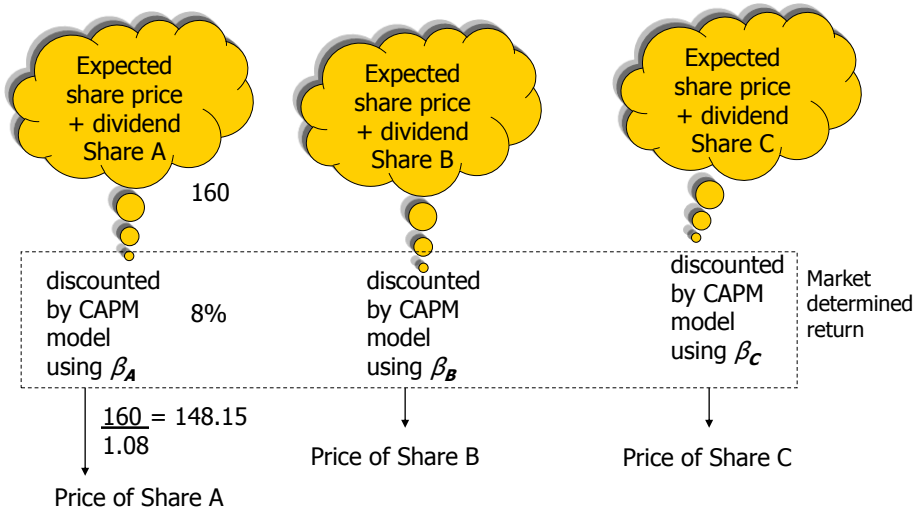
x = market returns

0.6027 = beta

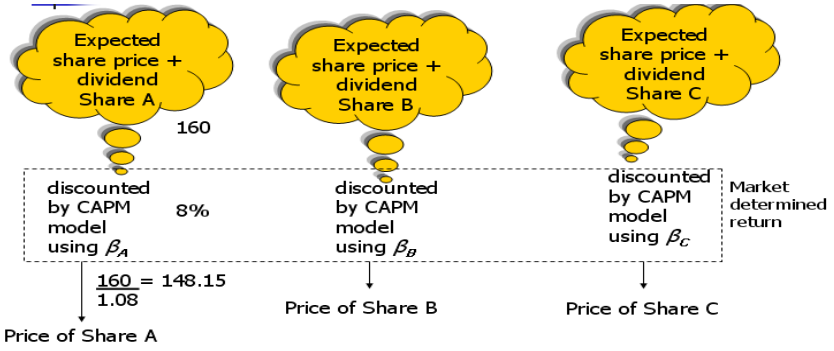
Web Sites for Beta

- http://finance.yahoo.com
  - Enter the ticker symbol for a "Stock Quote", such as IBM or Dell, then click GO.
  - When the quote comes up, select Key Statistics from panel on left.
- www.valueline.com
  - Enter a ticker symbol at the top of the page.
- Most stocks have betas in the range of 0.5 to 1.5.

Risk and Return in the Stock Market



Risk and Return in the Stock Market

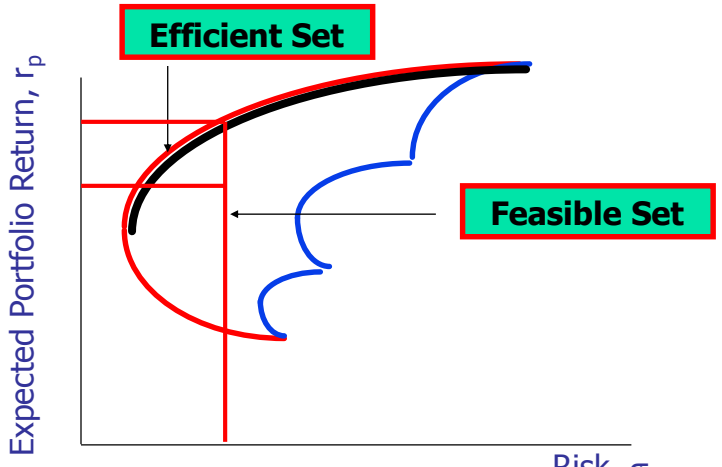


- A one period model (multiperiod a succession of single periods)
- From the CAPM model, returns should not have an *expected* return greater than its beta share of the market risk premium... $\beta_i \times E(r_m - r_f)$

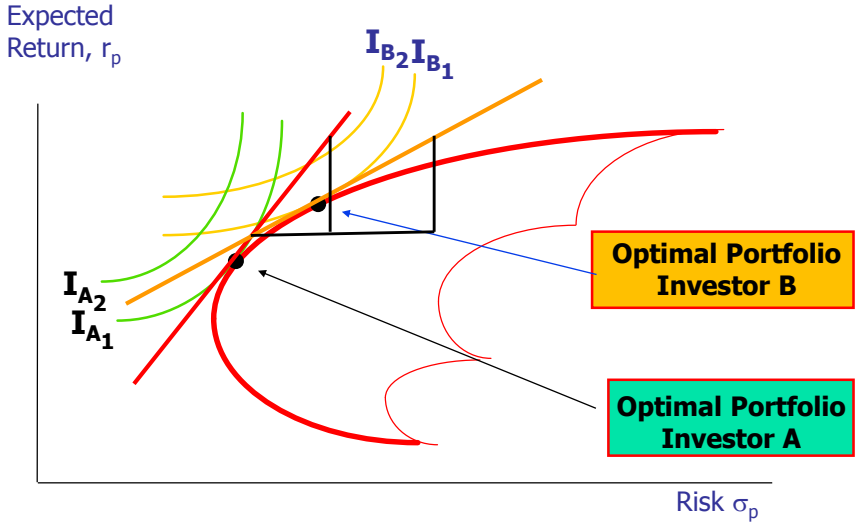
Feasible and Efficient Portfolios

- The feasible set of portfolios represents all portfolios that can be constructed from a given set of stocks.
- An efficient portfolio is one that offers:
  - the most return for a given level of risk, or
  - the least risk for a given level of return.
- The collection of efficient portfolios is called the efficient set or efficient frontier.

Feasible and efficient portfolios (risky assets) (Brigham & Erhardt)



Optimal Portfolios for individuals (Brigham & Erhardt)



What is indifference curves

- Indifference curves reflect an investor's attitude toward risk as reflected in his or her risk/return tradeoff function. They differ among investors because of differences in risk aversion.

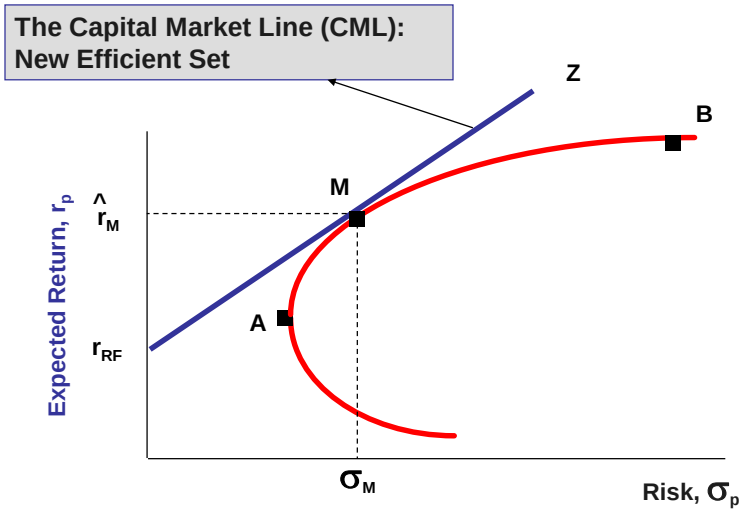
- An investor's optimal portfolio is defined by the tangency point between the efficient set and the investor's indifference curve.

Efficient risky portfolios

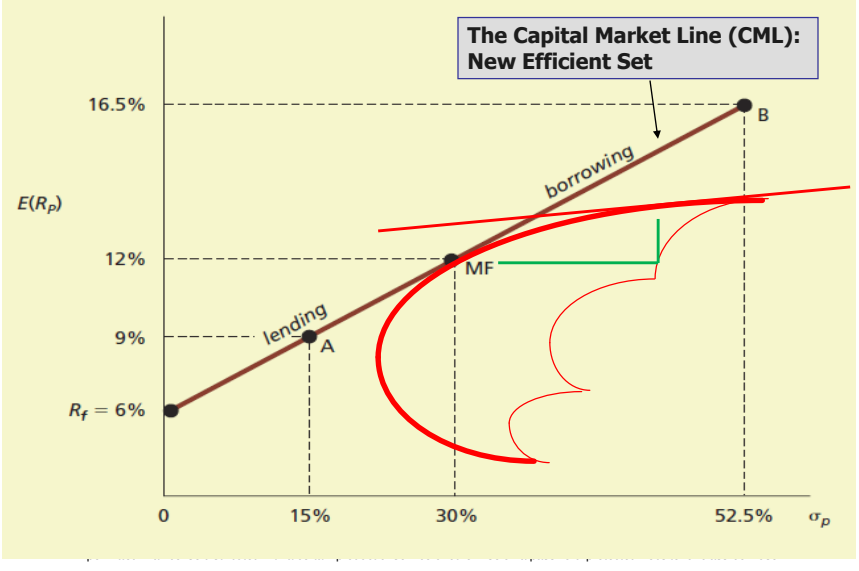
Suppose Stock A has an expected return of 6% and a standard deviation of 20%. Stock B has an expected return of 17% and a standard deviation of 50%. If the correlation between A and B is 0.4, what are the expected return and standard deviation for a portfolio comprised of 75% of stock A and 25% of stock B?

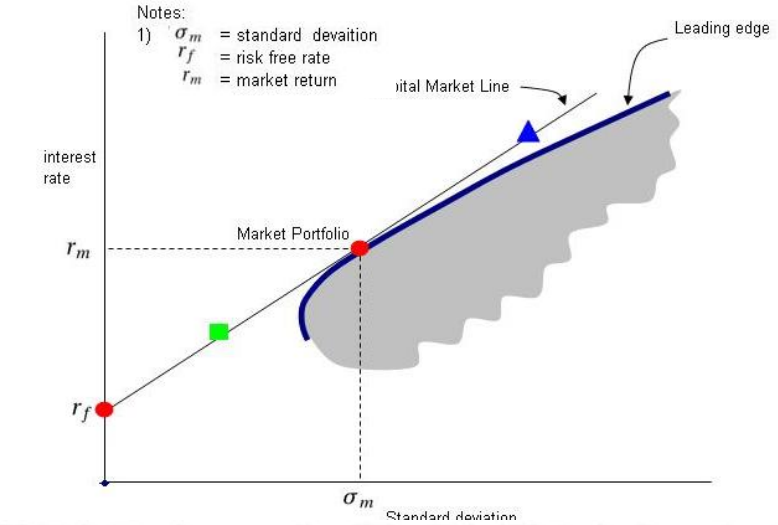
Attainable Set

Portfolios of Risky and Risk-Free Assets



Portfolios of Risky and Risk-Free Assets





2) The leading edge represents portfolios with the highest return for any given risk
3) All points between the two dots (the square) represent portfolios that combine the risk free investment and the market portfolio in differing proportions.
4) The triangle represents "leveraged investment" borrowing at the risk free rate and investing in the market portfolio.

Riskless borrowing and lending (Next 3 slides from Megginson) (I have $200 to invest with (1/3) probability of return)

Risky asset X → Three possible returns: -10%; 10%; 30% → Expected return = 10%; Standard deviation =16.3%

Risk-free asset Y → Return (r_f): 6% → Buying asset Y = Lending money at 6% interest

How would a portfolio with $100 (50%) in asset X and $100 (50%) in asset Y perform?

$100 Asset X
$100 Asset Y → Three possible returns: -2%; 8%; 18% → Expected return = 8%; Standard deviation =8.16%

Portfolio has lower return but also less volatility than 100% in X
Portfolio has higher return and higher volatility than 100% in risk-free

Riskless borrowing and lending (continued)

What if we sell short asset Y instead of buying it?
Borrow $100 at 6%
Must repay $106

Invest $300 in X Original $200 investment plus $100 in borrowed funds (implies 150% in X)

When X Pays –10%

$$\text{Net Return on \$200 Investment} = \frac{\$270 - \$106 - \$200}{\$200} = -18\%$$

When X Pays 10%

$$\text{Net Return on \$200 Investment} = \frac{\$330 - \$106 - \$200}{\$200} = 12\%$$

When X Pays 30%

$$\text{Net Return on \$200 Investment} = \frac{\$390 - \$106 - \$200}{\$200} = 42\%$$

Expected return on the portfolio is 12%. Higher expected return comes at the expense of greater volatility

Riskless borrowing and lending (continued)

The more we invest in X, the higher the expected return

The expected return is higher, but so is the volatility

This relationship is linear

Portfolio	Expected Return	Standard Deviation
50% risky, 50% risk-free	8%	8.16%
100% risky, 0% risk free	10%	16.33%
150% risky, -50% risk free	12%	24.49%

Finding the Optimal Risky Portfolio

- If investors can borrow and lend at the risk-free rate, then from the entire feasible set of risky portfolios, one portfolio will emerge that maximizes the return investors can expect for a given standard deviation.

- To determine the composition of the optimal portfolio, you need to know the expected return and standard deviation for every risky asset, as well as the covariance between every pair of assets.

The Capital Market Line (CML) Equation

$$E(R_p) = R_f + \left[\frac{E(R_m) - R_f}{\sigma_m}\right]\sigma_p$$

Intercept

Slope

Risk measure

What is the Capital Market Line?

- The Capital Market Line (CML) is all linear combinations of the risk-free asset and Portfolio M (market portfolio).
- The line connecting R_f to the market portfolio is called the Capital Market Line (CML)
- *CML* quantifies the relationship between the expected return and standard deviation for portfolios consisting of the risk-free asset and the market portfolio, using
- Portfolios below the CML are inferior.
  - The CML defines the new efficient set.
  - All investors will choose a portfolio on the CML.

Risk and Return in the Stock Market

- So how well does the Stock Market value shares?
-
- see Efficient Markets Hypothesis

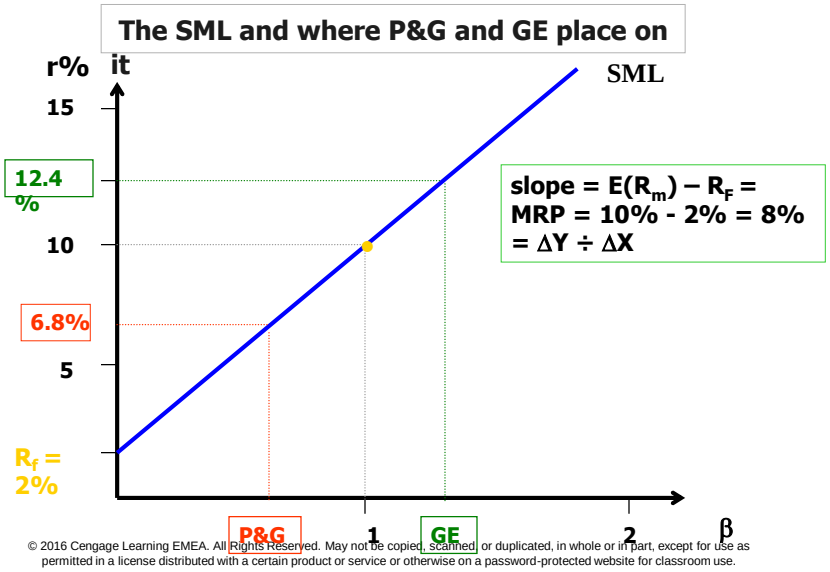
Risk and Return in the Stock Market

- Are there any other models that explain

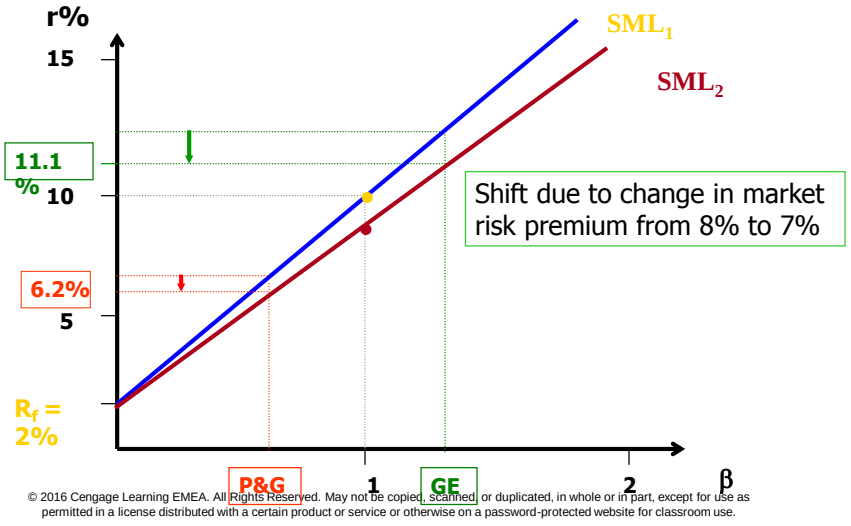
$$E(r_i) = r_f + \beta_i(E(r_m) - r_f)$$

- In practice, yes; in theory no!
- Fama French adds:
  - SMB a measure of size "small minus big"
  - B /M book to market ratio
  - HML "high B/M minus low"
  - what does this tell us? Not much?!!

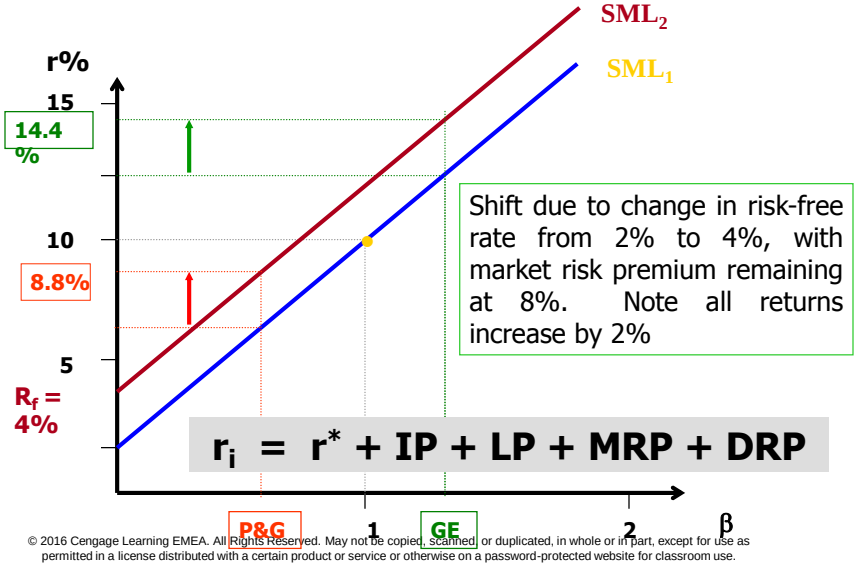
Using the security market line



Shifts in the SML due to a shift in required market return



Shifts in the SML due to a shift in the risk-free rate



Alternatives To CAPM

Arbitrage Pricing Theory

$$R_i - R_f = \beta_{i1}(R_1 - R_f) + \beta_{i2}(R_2 - R_f) + \beta_{i3}(R_3 - R_f) + \cdots + \beta_{in}(R_n - R_f)$$

Fama-French Model

$$R_i - R_f = \alpha + \beta_{i1}\left(R_m - R_f\right) + \beta_{i2}\left(R_{\text{small}} - R_{\text{big}}\right) + \beta_{i3}\left(R_{\text{high}} - R_{\text{low}}\right)$$

Have we met our goals

Are we knowledgeable on risk and risk management

And are we able to calculate:

- Expected return;
- Variance;
- Standard deviation;
- Coefficient of variation;
- For both stand-alone and portfolio risk.

Have met our goals

Are we able to:

Discuss portfolio risk, efficient portfolios and select an optimal portfolios

Determine market risk for individual companies